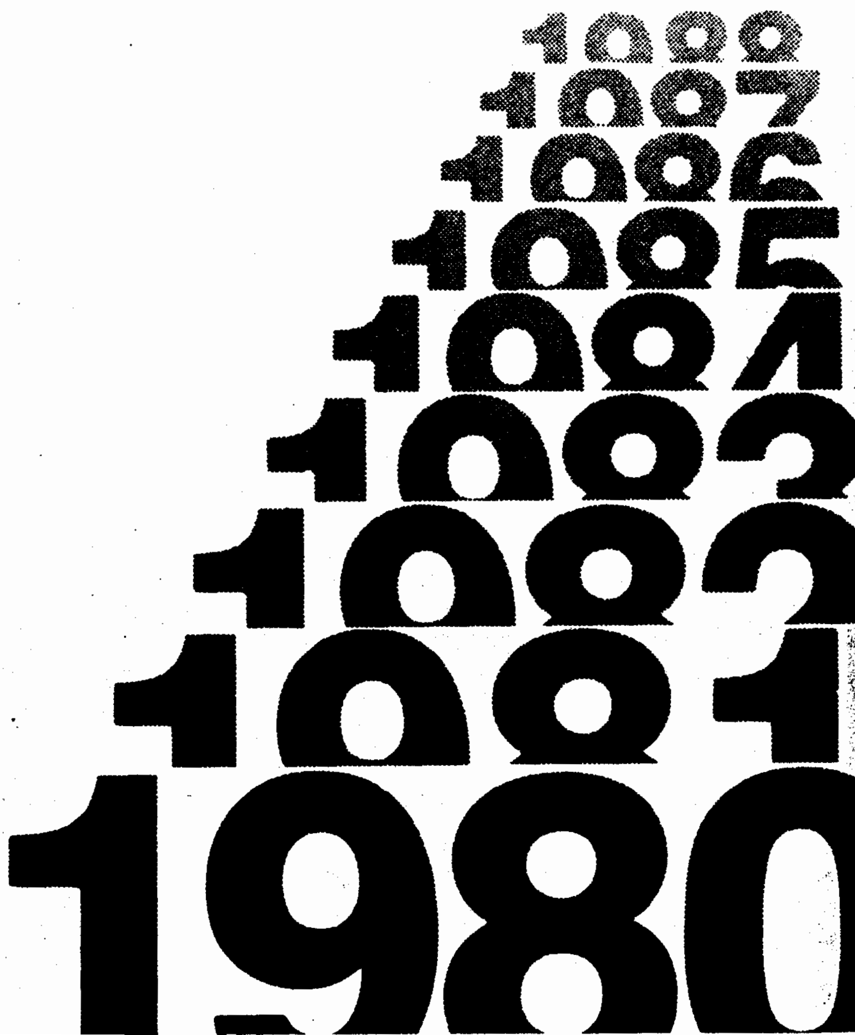
National Council of Teachers of Mathematics

# PRIORITIES IN SCHOOL MATHEMATICS

## Executive Summary of the PRISM Project



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# Preface

*An Agenda for Action: Recommendations for School Mathematics of the 1980s,* developed by the National Council of Teachers of Mathematics, has received a great deal of national attention. It appears that these basic recommendations for school mathematics in this decade are being considered carefully and implemented by those responsible for curriculum innovation as well as by publishers of mathematics textbooks at all grade levels.

The recommendations offered in *An Agenda for Action* represent a broad base of beliefs about objectives and priorities rather than the collective thought of a selected group of individuals. The Council conducted an extensive survey of the opinions of many sectors of society, both lay and professional, by means of a project entitled Priorities in School Mathematics (PRISM), which as funded by the National Science Foundation.

The PRISM survey of preferences and priorities may serve as a guide to the curriculum changes that mathematics educators and teachers are ready and willing to accept and implement. As such, they are a useful complement to *An Agenda for Action* in steering the mathematics curriculum during the 1980s. Indeed, this Executive Summary presents the eight recommendations in *An Agenda for Action* together with a summary of the related findings for each.

The National Council of Teachers of Mathematics is indebted to the members of the PRISM Project for their careful and detailed work and analysis: Alan R. Osborne, director; Jon L. Higgins; Peggy Kasten; and Marilyn N. Suydam. A special note of appreciation is due to Marilyn Suydam, who prepared this Executive Summary, which the Council is making available to all interested members of the profession.

A draft of this report was reviewed by Shirley Hill, LeRoy Dalton, Catherine Tobin, and Joe Crosswhite, members of the Mathematics Curriculum for the 1980s Committee, and by Jon Higgins and Peggy Kasten, members of the PRISM Project staff. Their comments are gratefully acknowledged.

MAX A. SOBEL, *President*
National Council of Teachers of Mathematics

# INTRODUCTION

The Priorities in School Mathematics Project (PRISM) was designed by the National Council of Teachers of Mathematics to collect information on current beliefs and reactions to possible mathematics curriculum changes during the 1980s. Knowledge of current beliefs can be useful in predicting which curriculum changes might be readily adopted and which ones might meet with resistance. Thus the data have a continuing usefulness as efforts are made to implement NCTM's *Agenda for Action* or seek other changes in the mathematics curriculum.

## The Survey of Preferences

The first component of PRISM was a survey of preferences for alternative content topics, instructional goals, resources, methods, provisions for particular groups of students, and ways of using calculators. Each of these was considered for nine content strands in the precollege curriculum: whole numbers; fractions and decimals; ratio, proportion, and percent; measurement; algebra; geometry; probability and statistics; computer literacy; and problem solving.

A pool of 660 strand items, plus 30 demographic items and 45 introductory items, was developed. Item-sampling techniques were used to allow a larger pool of items than would have been possible if a single instrument had been given to all persons. Each survey form contained either 110 or 120 items, plus 15 demographic and introductory items, with sets of items systematically distributed across forms.

The survey instruments differed on items pertaining to elementary and secondary school content, but most other items were similar across forms sent to teachers at all levels. The items were presented in clusters of 5, 10, or 15, leading from a common stem. One example of a portion of a cluster from the preferences survey follows.

> During the 1980s it may be possible to add to each classroom several different resources for teaching measurement. To what extent would you want to have each of the following?
>
> a. I would definitely want this.
> b. This might be nice to have.
> c. Undecided
> d. I'd rather not be bothered by this.
> e. I definitely would not want this.
>
> 1. A basic kit of measuring tools for each student
> 2. Films or videotapes showing basic measuring processes
> 3. Masters of worksheets and activities
> 4. Electronic measuring tools that show all measurements on a digital display similar to that of a calculator
> 5. Individual study materials for measurement

In many cases, teachers and teacher educators were given "generic" items that generalized across content strands. For the lay groups, who might not understand precise mathematical language, the items were simplified—or deleted when it was not possible to develop a parallel item. For instance, the lay samples did not receive items that pertained only to content topics. An example of a cluster from one of these forms follows.

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Full details on the procedures, the survey items, the data, and the analysis and interpretation of the data can be found in the Final Report, available from the ERIC Document Reproduction Service (Box 190, Arlington, VA 22210). The body of the Final Report and the two appendixes listing the survey items are included in ED 184 891; the remaining appendixes containing computer printouts of the data are in ED 184 892.

A committee is working on a curriculum guide for mathematics during the 1980s. The following statements were proposed by various committee members. Please react to each. Omit the item if you do not understand it.

a. I agree completely.
b. I tend to agree.
c. I am undecided.
d. I tend to disagree.
e. I strongly disagree.

1. Student worksheets or workbooks are included for drill and practice at the conclusion of each lesson.
2. Activities are included that anticipate the class being divided into small groups.
3. Only problems which students can answer quickly are assigned.
4. The introduction of calculators is postponed until students have learned both the meaning of, and paper-and-pencil procedures for, whole number computation.
5. Ideas or procedures are developed through real-life problem situations or activities.

### The Survey of Priorities

The second component of the PRISM study assessed priorities for curriculum change or for methods of addressing problems in mathematics education. Respondents were asked to judge the relative importance of curriculum alternatives, identifying points in the mathematics curriculum where change seems most needed or desired. Forced-choice items presented five alternatives to be rank-ordered in terms of the priorities for the 1980s. For some items, professional samples were given a set of five reasons to indicate why they assigned highest or lowest priority as they did. General factors and ways to approach problems in mathematics education were also considered. In all, professional samples responded to 92 items and lay samples to 84 items. In addition, 15 demographic and introductory items were included, as on the preferences survey.

One item from the preferences survey follows.

In the 1980s there will be a limited amount of money that can be spent for the development of new materials in the areas listed below. Please indicate the order in which you think the money should be spent by marking the answer sheets in the following way:

a = highest priority
b = second highest priority
c = middle-level priority
d = second lowest priority
e = lowest priority

Be sure to use each letter *only once* for the next five items.

1. Whole number computation
2. Problem solving in mathematics
3. Measurement
4. Fractions (concepts and computation)
5. Decimals (concepts and computation)
6. Consider the content area above that you ranked *highest* (marked with an "a"). Of the following five ideas, which best describes the reason you gave it highest priority?

a = We have fewer good materials to choose from in this area than in the other four areas.
b = This is a major problem area for many, many teachers.

c = It is absolutely crucial that all students develop skills in this area.
d = New ideas have been developed in this area that are not reflected in present materials.
e = The importance of this area will increase during the 1980s.

The specific focus of other items will be developed as major points from the surveys are presented.

### Populations Sampled

Many groups of people are involved in successful curriculum change. One intent of PRISM was to identify discrepancies in the patterns of curricular preferences and priorities held by such groups. Opinions were sampled and contrasted for nine populations, each with a stake in the directions that curriculum may take in the future. The first six were referred to as "professional" samples and the last three as "lay" samples. All were selected from lists obtained from appropriate organizations.

AT subscribers to the *Arithmetic Teacher*, a journal for elementary school teachers, and nonsubscribing teachers

MT subscribers to the *Mathematics Teacher*, a journal for secondary school teachers, and nonsubscribing teachers

JC junior college mathematics teachers

MA college teachers of mathematics (or mathematicians)

SP supervisors of mathematics

TE mathematics teacher educators

PR principals of elementary and secondary schools (who do not necessarily have mathematics education responsibilities, and so were categorized with the following samples as lay samples)

SB presidents of school boards

PT presidents of parent-teacher organizations

One problem of sample selection is indicated by the fact that only 32% of the AT sample taught in grades K-6, whereas 41% taught in grades 7-8 and 19% in grades 9-12. The MT sample consisted of 73% actually teaching in grades 9-12 and 7% in grades 7-8. Thus, the label "elementary school teacher" or "secondary school teacher" did not seem appropriate, and the samples are referred to as the AT or MT sample.

Over 10 000 preferences survey forms were mailed between September 1978 and February 1979; the return rate ranged from 15% to 60% for the nine samples, with an average of 29%. The priorities survey was mailed to 3750 persons in February and March 1979; the return rate ranged from 10% to 62%, with an average of 34%. The low response rate can be considered a limitation; however, since the intent was to obtain reactions from those who have an interest in the school mathematics program, it can also be considered as representative of those persons with such high interest.

The samples tended to be reasonably well-distributed across most demographic facets, including (as appropriate to the sample) number of years of teaching or administrative experience, type of community, supervisory level, and age of self and of children in school.

To the query about their level of satisfaction with schools, from 25% to 79% of the subjects indicated they were satisfied or very satisfied (see Figure 1). It is interesting to note that the principals (PR) and school board members (SB)—those responsible for broad

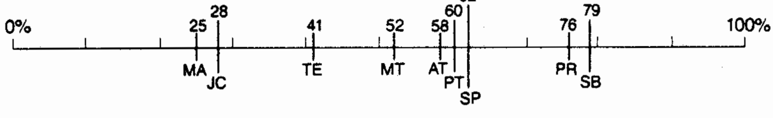



Figure 1. Percentages indicating satisfaction with schools.

decision-making about schools—were most satisfied, whereas junior college (JC) and college teachers (MA)—who face the problem of college entrants who need remedial help—were least satisfied. Classroom teachers (AT, MT) and parents (PT) clustered around the midpoint.

#### Introductory Items

Each respondent reacted to 15 introductory items on the preferences survey or 8 items on the priorities survey. Each item reflected an issue of particular concern in the current mathematics education community. Respondents were asked to indicate whether each should receive "much more emphasis," "somewhat more emphasis," "about the same emphasis," "somewhat less emphasis," or "much less emphasis" than it currently receives.

For about two-thirds of the items, "much more emphasis" or "somewhat more emphasis" was checked by at least 50% of the respondents. The range in level of support exceeded 30% in about half the instances. In Figure 2, the range of responses for some selected items is shown. Except for one instance, the mean level of agreement that each should be given more emphasis exceeded 50%; the exception is the last item, pertaining to calculators, which was included because the range of responses is so broad. The samples with the highest and the lowest level of support are indicated. (Six to nine samples responded to each item.) Among those items receiving little support for increased emphasis in the 1980s were formal

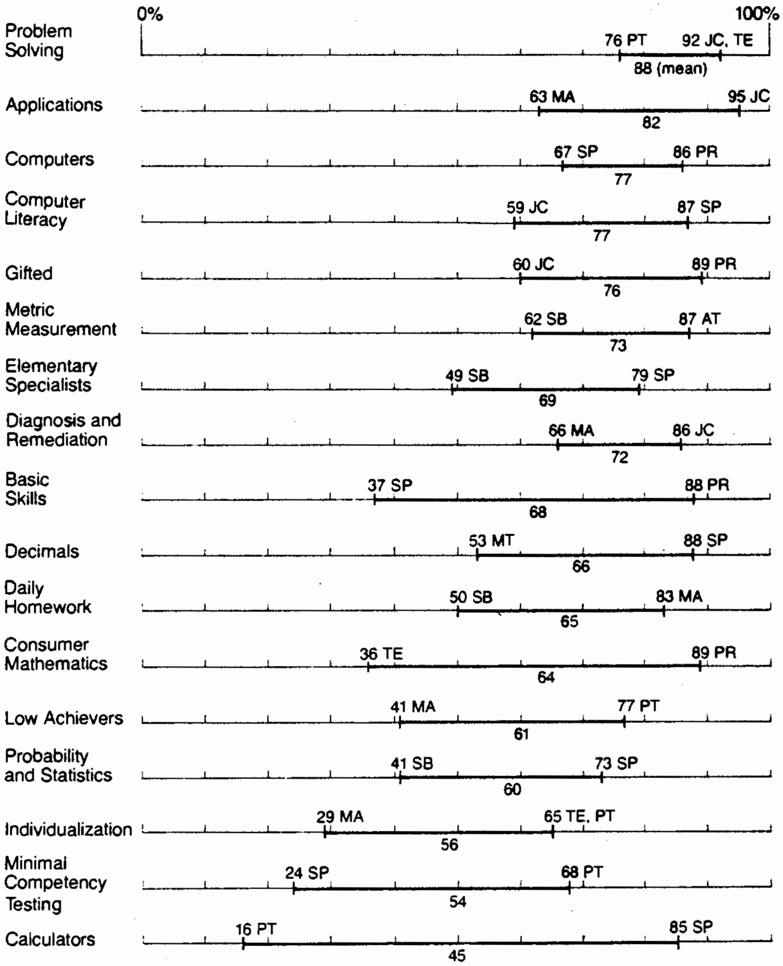



Figure 2. Range on selected introductory items.

axiomatic structures, norm-referenced testing, calculus in high school, proof, and large-scale curriculum development projects.

Of note in Figure 2 is the narrow, positive range for such issues as problem solving and diagnosis and remediation. The small discrepancy across groups could indicate that curricular changes pertaining to these issues would be viewed favorably. Such an attitude toward problem solving is encouraging as NCTM seeks to implement the first recommendation in *An Agenda for Action*, which proposes that problem solving be the focus of school mathematics in the 1980s. On the other hand, the range for such topics as basic skills and calculators is broad, indicating that proposed changes may not be as readily welcomed by some samples. Thus, plans to implement the third NCTM recommendation, which proposes the use of calculators (and computers) at all levels, could be more difficult. While supervisors (and teacher educators) supported the use of calculators, parents were negative about them and teachers were only slightly more enthusiastic.

Of course, there are problems of definition inherent in this set of items as well as differences in the perception of what constitutes a need for more emphasis. A case in point may be basic skills, where there is wide discussion of the meaning of the term; the sample of supervisors who would not give more emphasis to basic skills might have been responding from a different definition than the sample of, for instance, principals. Such attempts at interpretation of PRISM data can only be conjectural, of course.

## MAJOR POINTS FROM THE PREFERENCES SURVEY

PRISM generated a vast amount of data, and it is impossible to present or discuss every item in this summary. Therefore, only key points have been selected, using as criteria (a) levels of agreement or disagreement across and between samples and (b) the relationship to curricular changes proposed in NCTM's *Agenda*. The percentages of support indicate the sum of the "strongly agree" and "agree" responses. The data were first analyzed by the nine content strands; then items pertaining to the same point, but appearing in different strands, were synthesized. Because the same items are reviewed in these two contexts (within and across strands), there is some repetition of the selected points that follow, although an attempt was made to keep this to a minimum.

### Within Strands (Strand by Cluster)

*1. Whole Numbers*

- Five goals for teaching whole number computation received strong support from over 82% of the respondents; these related to consumer education skills, fundamental understandings, logical thinking, job qualifications, and the structure of mathematics. Clearly rejected as a goal for teaching whole numbers was "to preserve a traditional emphasis in the curriculum."

- Strong support (above 80%) was given for including estimation, mental calculations, specific strategies for solving word problems, and mathematical puzzles and games when teaching whole number content. Developing operations simultaneously and teaching such specific consumer skills as balancing a checkbook or computing best buys, were also well supported (by 72%).

- Resource books of real-life examples, masters of worksheets and activities, standardized practice tests, and individual study materials were strongly supported for teaching whole numbers, with other resources (including videotapes, calculators, and computers) supported by at least 65%.

- Eighty-two percent of the respondents indicated that calculators should not be used until after students have learned both the meaning of the whole number operations and paper-and-pencil algorithms for them.

- Support was strong for using calculators for checking and for doing a series of computations; it weakened, however, as the indicated computation was perceived to require skills involving traditional paper-and-pencil procedures.

• Over 90% indicated strong support for the idea that paper-and-pencil computational skills should be acquired before graduation from high school.

• The use of various physical materials for teaching whole number ideas was supported by 60% to 75%.

• Spending more than 50% of instructional time on drill with whole numbers was supported by 66%, whereas spending 50% of instructional time on individual study materials was supported by only 43% (see (Figure 3).

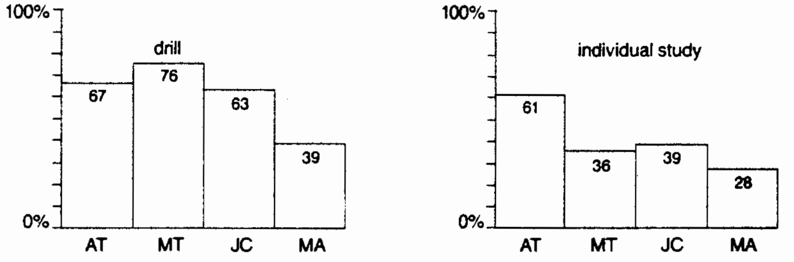



Figure 3. Percentages supporting spending 50% of time on drill or on individual study with whole numbers.

*2. Fractions and Decimals*

• Strong support (from over 80%) was given to four goals for teaching fractions, related to (a) their use in vocations, (b) consumer purchases, (c) illustrating basic mathematical ideas, and (d) providing solutions to algebraic equations.

• Strong support in the teaching of decimals was given to the first three goals supported for fractions, and also a fourth: their use in the metric system.

• Support by 73% to 83% was given to treating four topics on fractions and decimals in the elementary school: least common multiple and greatest common divisor as basic ideas related to fractions, estimation of the size of a dividend, decimals developed as a means of naming numbers between numbers, and fractions presented as answers to division problems (for example, 7/12 means 7 divided by 12).

• Support from 76% to 85% was given to treating the *same* four topics on fractions and decimals in the secondary school that received most support as elementary school topics.

• The four resources most desired for teaching fractions and decimals were (a) drill and practice materials, (b) masters of worksheets and activities, (c) resource booklets with applications, and (d) individual study materials.

• Two strategies for teaching fractions and decimals were given strong support: having worksheets for drill and practice on each lesson and using applications to develop operations with fractions.

• Only one item was given even moderate support (76%) from the set of items on when and to whom fractions should be taught; this approved the teaching of fractions with small denominators useful in vocations. There was very little support (15%) for delaying work with fractions until grade 7 or 8 or for omitting division of fractions except for very bright students. There was almost *no* support (1% to 3%) for teaching fractions only to college-bound students or for omitting fractions from the curriculum.

• Only one use of calculators to teach fraction and decimal ideas was strongly supported; it involved finding an area when dimensions were given to two decimal places.

*3. Ratio, Proportion, and Percent*

• Strong support (over 80%) was given to three goals for teaching ratio, proportion, and percent: to acquire consumer skills, to develop proportional thinking as a problem-solving technique, and to acquire skills for vocational applications. These three goals were practical; there was moderately strong support for four statements that were more "academic," such as "to demonstrate that ratios provide the foundations for a powerful reasoning process."

• Very strong support was given to the introduction of percent through merchandising and other real-life contexts.

• Six resources for teaching ratio, proportion, and percent were supported by over 78%: resource books of applications, master copies of activities and worksheets, books of laboratory experiments, manipulative materials, short films and videotapes, and individualized study materials.

• Support by over 78% was given to three strategies for teaching ratio, proportion, and percent: student worksheets for drill and practice, physical experiments in a laboratory setting, and projects for assignment to individuals or teams of students.

• There was moderately strong support for only one item on when and to whom to teach percent: 64% agreed that mastery of percent problems should be a condition for high school graduation (see Figure 4).

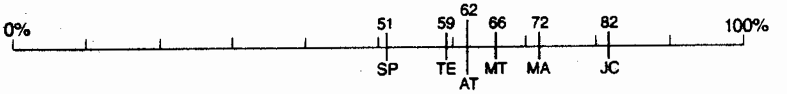



Figure 4. Percentages supporting mastery of percent problems as a graduation requirement.

• Teaching variation in science classes, not introducing ratio and proportion until grade 9, and teaching all three types of percent only to bright students were strongly rejected (66% to 87%).

• Only one use of calculators for teaching ratio, proportion, and percent—checking answers—was strongly supported (92%).

*4. Measurement*

• Six goals for teaching measurement were strongly supported by over 80% of the respondents. These goals involved acquiring skills for living in today's world, for other school work, for use in the home, for estimation, for jobs, and for using specific tools.

• Four topics received strong support as measurement content for elementary school mathematics: the metric system, the use of measurement devices, estimation, and the use of both standard and nonstandard units of measure.

• For measurement content for all students in grades 7–12, three items received strong support: the metric system, estimation, and the multiplication and division of units.

• The four resources for measurement most strongly supported (over 80%) included resource books with problems, masters of worksheets or activities, student books with experiments, and a basic kit of measuring tools.

• Four other resources for measurement were supported by about 78%. Thus it is apparent that the samples wanted almost any resource (as was also true in other content strands); they failed to support only two electronic resources.

• The listed teaching strategies for measurement were also well accepted, with above 70% support for 7 of 10 strategies. Highest support was given to (a) assignments including projects and (b) worksheets for drill and practice at the conclusion of each lesson.

• Over 80% felt that work on measurement should be taught at every level from K–8, and over 70% indicated that measurement should be a strong focus of general or consumer mathematics courses.

• Only two calculator uses for teaching measurement received strong support: checking answers and computing a volume problem.

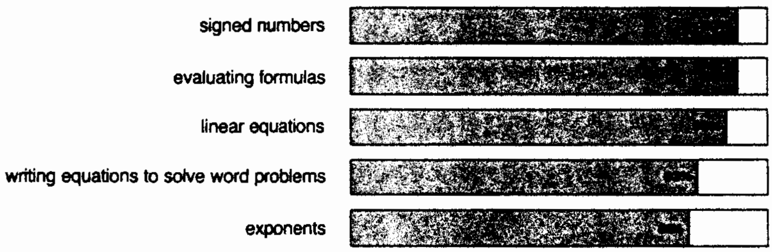
*5. Algebra*

• Strong support (over 80%) was given to four goals for teaching algebra; these concerned applying mathematics, building background for taking more mathematics, gaining vocational skills, and preparing for college. The remaining goals were accepted by over 60%—except for "assuring adequate scientific manpower," supported by only 44%.

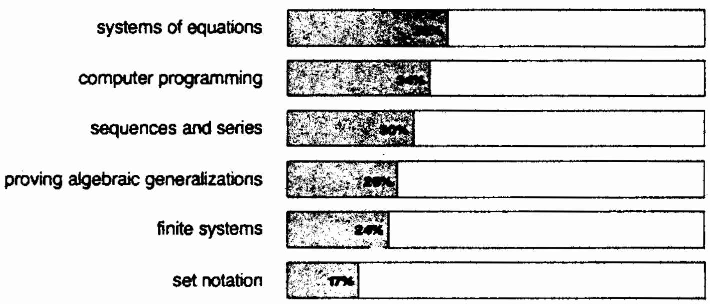
• Support was strongest (above 70%) for including four of fifteen algebraic topics in the elementary school curriculum: solving number sentences, making generalizations about number patterns, writing equations to solve word problems, and inequalities.

• Respondents from the AT, SP, and TE samples did *not* support including four of the fifteen algebraic topics in the elementary school curriculum, including finite systems, set notation, and computer programming.

• Strong support was given for including five algebraic topics in the curriculum for *all* students:




• The samples did *not* support teaching all students the following algebraic topics:




Thus, the list of algebraic topics to be taught to all students would not go beyond those topics that have been in junior high textbooks for years.

• For college-bound students not majoring in mathematics or science, six algebraic topics were given moderate support (59% to 79%): probability functions, mathematical models, exponential and logarithmic functions, trigonometric functions, matrix algebra, and finite mathematics.

• Support is strongest (above 75%) for having four resources available for teaching algebra: booklets with applications problems, masters of worksheets and activities, booklets of games and activities, and physical materials and equipment for laboratory experiments.

• Support was strong (above 79%) for having instructional materials for algebra that emphasize problems arising in the social or natural sciences, having worksheets for drill and practice, and inferring algebraic ideas from arithmetic patterns.

• Only two items about the type of algebra course to be offered received support (at the 70% level): a special algebra course for vocational students and different courses for students with different interests and abilities.

• Totally rejected was the idea that formal work with algebra should be dropped from the curriculum.

• Using calculators for checking answers in algebra was the only use of calculators that was strongly supported.

• Using calculators during an algebra test was accepted and rejected by almost equal percentages.

*6. Geometry*

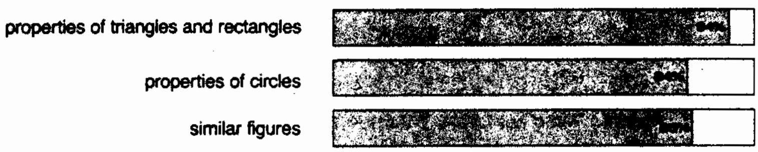
• Four goals for geometry received strong support (over 80%): to develop logical thinking abilities, to develop spatial intuitions, to acquire the knowledge for further study, and to learn to read and interpret mathematical arguments.

• Job and consumer skills were not ranked as high for geometry as for some other strands.

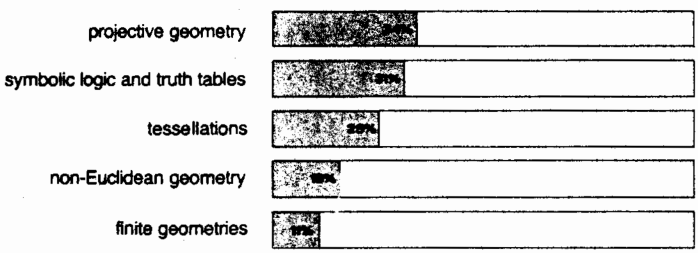
• Four geometric topics were strongly supported for inclusion in the elementary school curriculum: properties of triangles and rectangles, parallel and perpendicular lines, symmetry, and similar figures.

• Opposition was strong to the inclusion of "logical reasoning principles including axioms and proofs" in the elementary school curriculum.

• Strong support was given to the inclusion of three geometric topics in the secondary school curriculum for *all* students:




Five topics were given little support:




It would appear that minimal knowledge of minimal geometry is all that is considered necessary for students to learn.

• For college-bound students not majoring in mathematics or science, only two geometric topics were given moderately strong support (77% for each): coordinate geometry and straightedge and compass constructions.

• Support was above 69% for all except one resource for teaching geometry. Strongly accepted (above 80%) were resource books of applications, masters of worksheets and activities, and short films or videotapes showing basic geometric concepts.

• Only one teaching strategy for geometry received strong support (83%): student worksheets for drill and practice to be used at the conclusion of each lesson.

• The availability as an elective of a full-year course in applied geometry and the belief that intuitive geometric concepts are at least as important in grade 1 as number concepts were each supported at a moderately strong level (71% and 65%, respectively).

• Over 60% did not support three items: (a) abolishing separate courses in geometry in favor of integrating geometric content in other courses; (b) considering the geometric topics presently taught in elementary schools to provide adequate minimum knowledge for high school graduation; and (c) not teaching geometric topics until grade 7.

• The use of calculators was given strong support only for two problems with triangles.

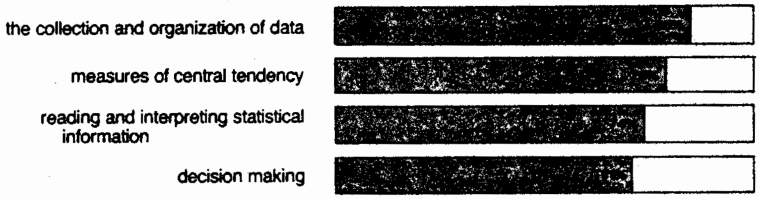
*7. Probability and Statistics*

• Five goals for probability and statistics were strongly supported (over 80%): using data in other subjects, dealing with statistical information as consumers, organizing data in easily interpretable forms, dealing with estimation and approximation, and applying mathematics in other disciplines.

• Two statistics topics were strongly supported for inclusion in the elementary school curriculum: collecting and organizing data, and reading and interpreting statistical information.

• Rejected for inclusion in the elementary school curriculum was calculating probabilities of compound and conditional events.

• Over 70% considered four probability and statistics topics appropriate for *all* secondary students:




• Four probability and statistics topics considered appropriate (by 66% to 71%) only for college-bound students were (a) curve fitting and prediction, (b) probability distributions, (c) combinations and permutations, and (d) calculating probabilities of compound and conditional events.

• Eleven of fifteen resources for probability and statistics were supported by over 75%; these ranged from books of applications to materials for use with computers. Thus, as for other content strands, most resources were seen as desirable.

• Three strategies for teaching probability and statistics were strongly supported: materials with real-world data, experiments, and problems from the sciences.

• Reading formal presentations before doing classroom activities on probability and statistics was rejected by 58%.

• Offering probability and statistics as an elective course was supported by 77%.

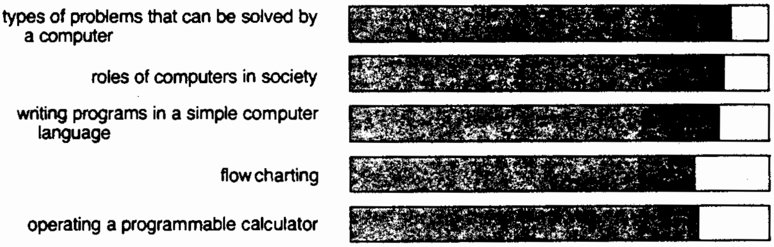
• Rejected (by over 65%) were making probability and statistics a required course for ninth graders, considering them only as enrichment topics, or using them to replace work with fractions in grades 6–8. There was rejection (over 85%) of the idea that either probability or statistics belongs in the curriculum but not both.

• Three uses of calculators for teaching probability and statistics were strongly supported (over 80%): doing homework, calculating the probability that several events will occur in a certain sequence, and taking a test.

*8. Computer Literacy*

• Two goals for computer literacy were given strong support (over 80%): to develop logical thinking abilities and to prepare for the future. Moderately strong support (over 70%) was given to two other goals: to understand the capability of the computer to provide access to large bodies of information, and to introduce alternative techniques for solving problems.

• More than 80% of those sampled believed that five computer literacy topics should be included in the curriculum:



• The resources for computer literacy supported most strongly (above 80%) were mini-computers and terminals connected to a large computer.

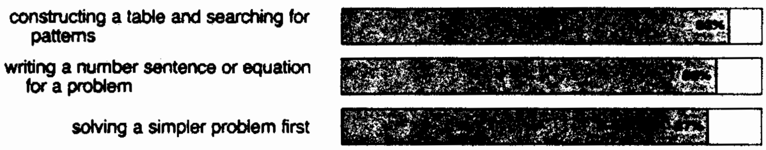
• Field trips to observe computers in use were supported by 74%. Four other items were given moderate support (63% to 68%): engaging in individual study projects, devoting 50% of instructional time to writing computer programs, providing detailed notes for the teacher, and using computer-assisted instruction in a tutorial mode.

• Support for items about when and to whom computer literacy topics should be taught were less strongly supported than items in other clusters. Highest support (68%) was given to integration of topics within the curriculum from grades K-12.

*9. Problem Solving*

• Five of ten goals for problem solving were strongly supported by 84% to 95%. These pertained to developing methods of thinking and logical reasoning, acquiring skills needed "for today's world," acquiring techniques vital to a well-rounded education, developing creative thought processes, and applying recently taught ideas. Four other goals were supported by 69% to 79% of the respondents.

• The teaching of three problem-solving techniques received strong support (87% to 92%) at both elementary and secondary level:




Receiving almost as much support (over 74%) were drawing a picture and guessing-and-testing. Using calculators or computers to generate possible answers, and then checking, was opposed at both levels. (Respondents gave moderate support for having calculators and computers available for problem-solving work, but it was not apparent just what they would have the students do with them.)

• Four of fifteen resources for teaching problem solving were strongly supported: resource guides to real-life problems, materials for modeling problems and solutions, and supplementary materials with many additional problems; in-service training for teachers was also strongly supported. Six other resources were supported by over 67%.

• Three strategies for teaching problem solving were strongly supported: problem assignments designed to challenge students to think, projects involving real-life situations for individuals or teams, and using problems to introduce mathematical topics.

• Both professional and lay samples strongly disagreed with the statement that "only problems which students can answer quickly" should be assigned.

• Two items on when and to whom problem solving should be taught received support by 79%: (a) teaching students to find problems within situations and (b) including short problem-solving units after each mathematical topic is taught. Seven items from this cluster were not accepted by large percentages (45% to 97%).

• Support was minimal (59%) for offering an interdisciplinary problem-solving course.

**Across Strands**

*1. Drill and Practice*

• Samples disagreed about whether more than 50% of instructional time should be devoted to drill and practice. Figure 5 indicates this—and also that some samples gave moderately strong support to this percentage of time despite research evidence that this is likely to be detrimental to achievement. Only the SP and TE samples were strongly opposed; they tended to be less supportive of many items pertaining to drill.

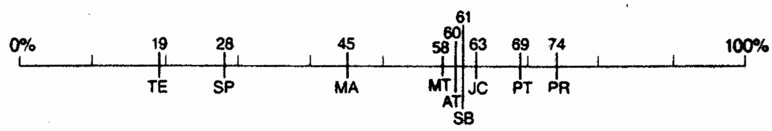



Figure 5. Percentages supporting increased time on drill and practice across content strands.

• Worksheets for providing practice at the conclusion of each lesson were given strong support (above 80%) for five content areas. Providing teachers with master copies of worksheets also received strong support for seven content areas in the elementary school, and only slightly lower support at the secondary school level.

• Providing drill and practice items in standardized test formats was strongly supported (82%) for whole numbers and moderately supported (75%) for problem solving by the AT, MT, JC, and MA samples. But the SP and TE samples gave the idea only minimal support.

• Mastery of whole number computational skills before graduation from high school received very strong support (85% to 100%) from all samples. Mastery of percentage problems before graduation received a similar level of support (81%) from lay samples, but more moderate support (64%) from professional samples.

2. *Applications*

• The need was noted to increase the amount of emphasis for applications of mathematics by 81% of the respondents. The level of support for mathematics and careers was 65% and for mathematics for consumers, 61%.

• Lay samples also gave very strong support (above 95%) to the importance of applications for solving problems in everyday life, gaining skills necessary for employment, and making consumer decisions. Strong support (84%) was also given to the importance of mathematics in preserving student options on career and vocational choices.

• There was strong support across all samples for mathematics course goals related to applications.

• Consumer needs as a goal of teaching particular aspects of mathematics were strongly supported for all areas except geometry, which apparently is not so strongly seen as related to consumer needs.

• Support for vocational or career goals was very strong (over 80%) for whole numbers; decimals and fractions; ratio, proportion, and percent; measurement; and algebra. Such goals received only minimal to moderate support for probability and statistics, computer literacy, and geometry, however. Respondents may feel that a smaller percentage of students will enter careers or vocations employing these skills.

• Resource booklets containing applications were strongly supported by over 80% of all samples for the eight content areas where they were suggested. However, resource books with problems and applications designed to appeal to special groups (for instance, girls or ethnic minorities) were given very little support.

• The use of applications as a context for instruction was given consistently strong support.

• Simulation as a teaching method was given moderately strong support by the AT, SP, and TE samples, but was only minimally supported by the MT, JC, and MA samples.

• Support for incorporating elements of consumer mathematics generally ranged from very strong (97%) to moderately strong (67%), depending on the specific content involved. One exception pertained to the inclusion of probability and statistics in general or consumer mathematics courses (see Figure 6). That probability and statistics is necessary for *all* students as consumers in today's world apparently has not yet reached wide acceptance.

3. *Individualization*

• Increasing the emphasis on individualization during the 1980s was given moderately

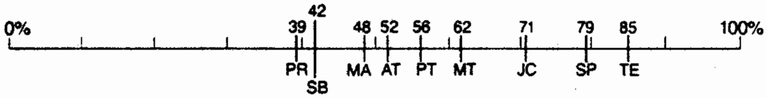



Figure 6. Percentages supporting inclusion of probability and statistics in general or consumer mathematics courses.

strong support by the AT, SP, TE, PR, SB, and PT samples. Little support was given by the MT, JC, and MA samples, however.

- All groups gave strong support (78% to 92%) for providing teachers with individual study materials for classroom use.
- Very little support was given to devoting more than 50% of instructional time to student use of individual study materials; however, the AT sample gave minimal support for the idea when used with whole numbers and measurement.
- Increasing emphasis on mastery learning received moderately strong support from AT and MT samples, but slightly less support from SP and TE samples. Specifying competency levels in instructional materials received very strong support (92%) from lay samples.
- Instructional materials with specific objectives, criterion-referenced testing, and other aspects of a mastery learning or individually paced model were given moderately strong support (above 63%) by all professional samples except the MA sample, which gave very little support.
- Individual study by computer-assisted instruction was given moderately strong support (64% to 72%).

*4. Differentiated Programs for Special Groups*

- When the lay samples were asked how many years of high school mathematics should be required, there was strong support for requiring at least two years of mathematics in grades 9–12 for *all* students, and almost half supported four years of mathematics for college-bound students (see Figures 7 and 8).

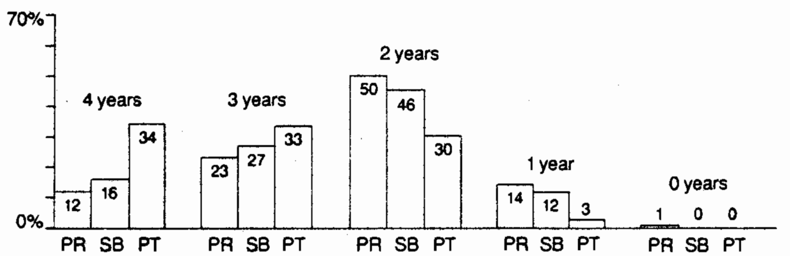



Figure 7. Years of mathematics supported for all students.

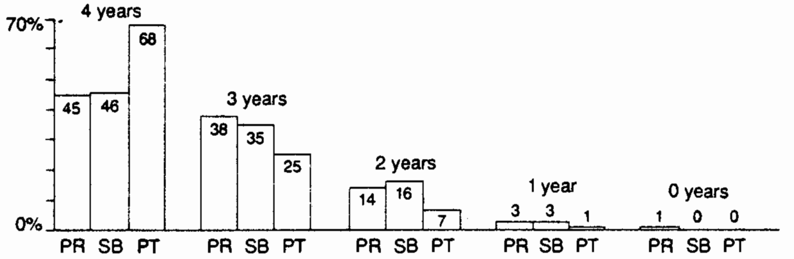



Figure 8. Years of mathematics supported for college-bound students.

- Increased emphasis for gifted students was supported by over 75% of most samples.
- Support for a senior-level probability and statistics course for students of high ability

was moderate from most professional samples (62% to 67%) except for the AT sample (54%) and the TE sample (46%). The PR and SB samples supported it at the 75% level, and the PT sample was lower (56%).

• The need for increased emphasis on women was supported by 53%, but 40% indicated that women should receive about the same emphasis as now.

• Different problem-solving courses for girls were strongly rejected (by 94%), and very little support (only 47%) was shown for providing resource books with problems that appeal to girls.

• Increased emphasis for minorities was supported by only 39%, and 50% felt that minorities should receive about the same emphasis as now.

• Increased emphasis for low achievers received moderately strong support (63% to 77%) from all samples except MA, SP, and TE, where support was minimal (41% to 55%).

• Teaching problem solving, division with fractions, and the three types of percent only to gifted students were each rejected by 80% to 90%.

• Extensive use of the computer in at least one mathematics course for college-bound students was given moderately strong support (64%) by the lay samples.

*5. Use of Calculators*

• Professional samples in general were much more supportive of increasing emphasis on calculators than were lay samples. Thus, just over 50% of the AT and MT samples and 85% of the SP sample would increase emphasis on calculators, but only 16% of the PT sample would do so (see Figure 9).

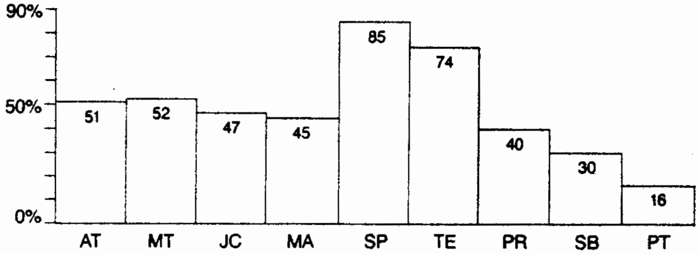



Figure 9. Percentages supporting increased emphasis on calculators.

• The use of calculators to develop ideas and concepts was supported by slightly less than 85% of the SP and TE samples; acceptance by the other samples was lower, averaging from 43% to 56%.

• The use of calculators to help children learn basic facts was given very little support; this apparently reflects the belief that basic facts should be learned before calculators are used, despite research evidence that pupils can learn basic facts while using calculators.

• Using calculators to learn why an algorithm works received moderate support (65%) (see Figure 10).

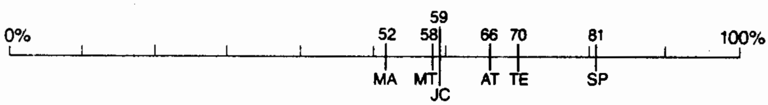



Figure 10. Percentages supporting the use of calculators to learn why an algorithm works.

• Checking answers was a noncontroversial use of calculators, ranking first among the possible uses of calculators in almost every area, with support from 84% to 97%.

• The use of calculators for solving word problems was strongly supported by the SP and TE groups and given moderate support by the AT, MT, JC, and MA samples. However, the lay samples were not as supportive of the idea (see Figure 11).

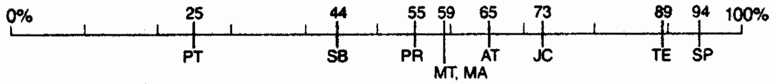



Figure 11. Percentages supporting the use of calculators for solving word problems.

• Using calculators for doing homework was supported by more than 85% of the SP and TE samples, and opposed by more than 70% of the PT sample (see Figure 12).

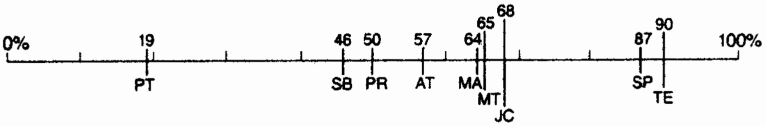



Figure 12. Percentages supporting the use of calculators for doing homework.

• Support for the use of calculators to do homework depended on the content area involved. It was strongest for probability and statistics and for ratio, proportion, and percent.

• There was very little support for using calculators to take tests except in probability and statistics. In general, the PT sample expressed strong disapproval (only 12% agreed, while 74% disagreed).

• Having the use of a calculator to solve equations was given strong support for geometric formulas but minimal support for algebra.

• Using a calculator to compute area was given moderate to strong support; support was strongest when the problem was more complex or when decimals were involved.

• Using a calculator to do a chain of calculations involving different operations was perceived favorably by all samples (77% to 95%).

• Using calculators in making graphs was given moderately strong support (70% to 85%) by the JC, MA, SP, and TE samples. However, there was lower support for this idea by the AT and MT samples.

• The use of calculators in trigonometry was given very strong support by all professional groups.

• Classroom availability of four-function calculators and programmable calculators was supported by more than 70% of all samples, but giving instruction for using a four-function calculator received only minimal support (45% to 51%) by teacher samples, with the SP and TE samples higher (70% and 81%, respectively).

• Very few believed that calculators should be used instead of paper-and-pencil algorithms (see Figure 13).

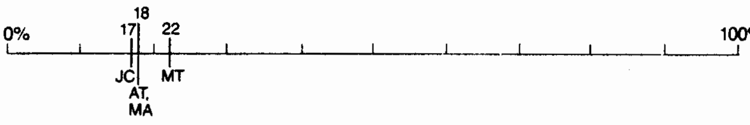



Figure 13. Percentages supporting the use of calculators in place of paper-and-pencil algorithms.

Rather, it was felt that their use should be postponed until after pencil-and-paper algorithms are learned (see Figure 14).

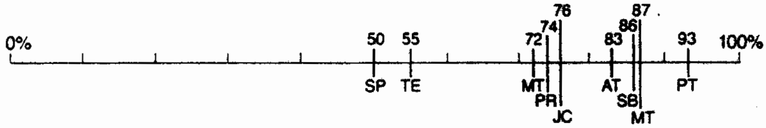



Figure 14. Percentages supporting the use of calculators only after pencil-and-paper algorithms are learned.

• Requiring students who have not learned paper-and-pencil computation by the end of grade 8 to take a calculator course in grade 9 received minimal support (see Figure 15).

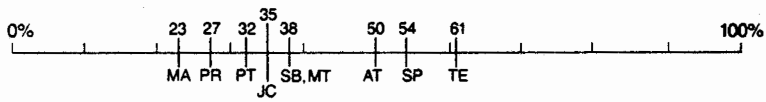



Figure 15. Percentages supporting a calculator course at the end of grade 8 for those who have not learned paper-and-pencil computation.

• The use of calculators to allow slower students "to keep up with the rest of the class" was generally opposed (only 12% to 44% approved of this use of calculators).

• For finding the sum of several items, the use of calculators was strongly supported (80%). But the use of calculators for subtraction was *not* favored.

• The use of calculators for multiplication generally received minimal support (above 50%) unless the problems were perceived as easy. There was similar lack of enthusiasm for using calculators for division.

• The *general* idea of using calculators with special displays or capabilities was given moderate to strong support by the SP, TE, and PR samples. However, results were divided when specific features were suggested.

*6. Use of Computers*

• Nearly 75% of the professional samples and 80% of the lay samples believed that the use of computers and other technology should be increased during the 1980s; 78% indicated that the emphasis on computer literacy should be increased. Further analysis indicates that the pattern for the two items is similar, with the largest percentages selecting the "somewhat more emphasis" option (see Figure 16). (This is characteristic of results on many items: people less often committed themselves to the extreme positions.)

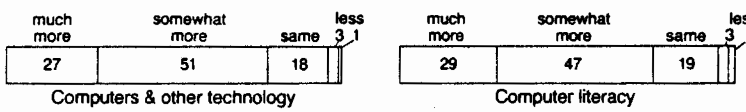



Figure 16. Percentages supporting increased emphasis on computers and on computer literacy.

• Instructional materials for computers that received moderately strong support included materials for individual projects (68%), workbooks with algorithms simulating computer processes (63%), detailed notes for teacher presentations (63%), and probability and statistics materials for use with computers (76%).

• Flowcharting and writing computer programs in BASIC were strongly supported (82% and 88%, respectively). However, other computer languages received much less support (31% to 57%).

• Almost no one (23%) believed that computer programming should be introduced in the elementary school, and very few in the professional samples believed that the ability to write programs should be a requirement for high school graduation (see Figure 17).

• Sixty-five percent of the lay samples supported the idea that at least one course in mathematics for the college-bound student should make extensive use of the computer.

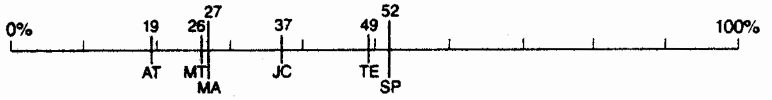



Figure 17. Percentages supporting the ability to write computer programs as a graduation requirement.

• Teaching about the roles of computers in society was strongly supported (89%). Although less concern was noted for teaching about privacy and security issues, these still received moderately strong support (67%).

• Studying about the types of problems computers can solve received very strong support (91%), while the goal of introducing alternative techniques for solving problems was approved by only 70%.

• Requiring a computer literacy course of all students was given minimal support (53%) by the professional samples and essentially no support (34%) by the lay samples. However, lay samples did give moderately strong support (79%) to the integration of computer literacy topics within the existing K-12 mathematics curriculum.

• Respondents were divided about whether computer courses should be strictly elective, with 35% favoring and 40% opposing.

• Some support (58%) was given to requiring interaction with computers as early as the primary grades.

• The idea that knowledge of computers is only needed by specialists was strongly opposed by 89%.

• Having computers or computer access for students was given strong support (95%) at the secondary school level and moderately strong support (77%) at the elementary school level. Strong support (84%) was shown for having several small, personal computers for each class.

*7. Estimation and Approximation*

• Development of estimation skills and experience in dealing with estimation and approximation were strongly supported (above 82%) as goals for measurement and for probability and statistics.

• Support for including estimation was strongest with measurement and whole number content.

• Approximation techniques in algebra were given minimal support.

*8. Laboratory Activity-based Approaches*

• Little support (48%) was given to increasing the emphasis on mathematics laboratories in the 1980s. An additional 34% opted for not changing the amount of emphasis.

• Support for introducing basic ideas through laboratory investigations or experiments with materials varied with topic, but were supported by most samples (see Figure 18).

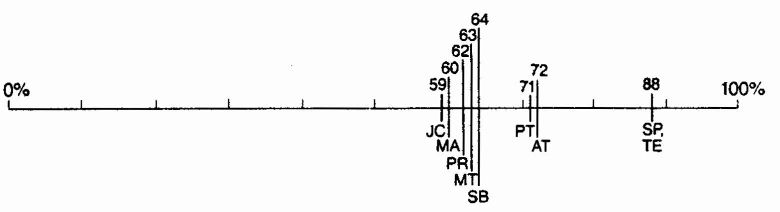



Figure 18. Percentages supporting the introduction of ideas through laboratory investigations or experiments.

• Lay samples gave stronger support (93%) to the use of physical materials and models than did professional samples (who varied from 62% to 83%, depending on the topic).

• The need for providing measuring devices as resources for fractions and decimals, geometry, and measurement was given moderately strong support (73% to 81%). However, support was far less for electronic measuring devices.

- Booklets of experiments for the three content areas that were considered (probability and statistics; ratio, proportion, and percent; and measurement) received strong support (above 80%).

- Having students work in small groups to solve problems was given a higher degree of support (73%) than dividing the class into small discussion groups; however, the level of support for such groups varied across samples (see Figure 19).

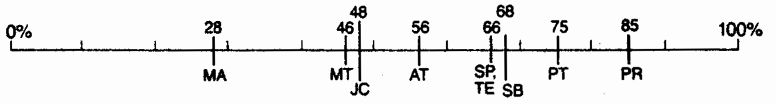



Figure 19. Percentages supporting small discussion groups.

*9. Out-of-Class Activities and Projects*

- Instructional materials that include activities requiring students to go outside the classroom were given moderately strong support by all samples. The MA sample was least positive about using out-of-class activities, while the AT sample was the most positive (see Figure 20).

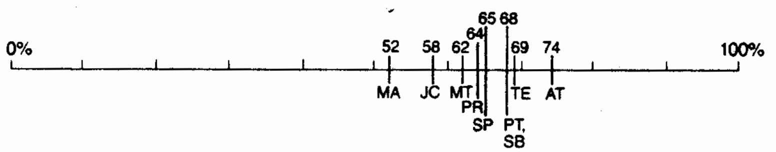



Figure 20. Percentages supporting out-of-class activities.

- Probability and statistics, measurement, and computer literacy were perceived as more suitable for out-of-classroom activities than whole numbers; geometry; ratio, proportion, and percent; or problem solving.

- The TE and MA samples were least supportive of developing ideas through long-term real-life projects (59%); the other samples were in closer agreement at approximately the 68% level.

- There was moderately strong support for project work in ratio, proportion, and percent; geometry; and probability and statistics. Project work for algebra and for fractions and decimals was not supported.

- The JC and MA samples strongly favored increasing the amount of homework, while other samples gave weaker support (see Figure 21). However, 84% of the lay samples indicated the desirability of text materials that include daily homework problems.

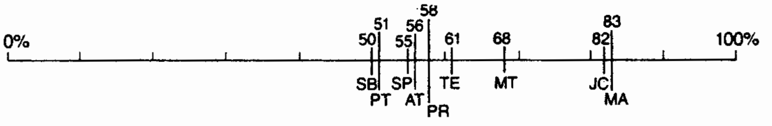



Figure 21. Percentages supporting increased emphasis on homework.

*10. Reading and Textbooks*

- "Learning to read mathematics" received moderately strong support (70% to 78%) as a goal for whole numbers, algebra, and problem solving.

• **Reading** a formal presentation of basic ideas *before* trying classroom activities was **generally opposed** by all samples.

• **The** availability of special materials with minimal reading requirements was given **moderately strong support** by the AT, SP, TE, and PR samples. However, the MT sample gave **weaker** support, and the JC and MA samples tended to oppose the idea.

• **De-emphasizing** reading by presenting problems orally or with pictures and charts was **not supported** (only 28% in favor). Another item proposing that reading be de-emphasized in textbooks and other materials was opposed by the SB and PT samples (69%).

• **Textbook modules** for teaching appropriate problem-solving strategies at every grade level **was supported** by 76%.

• **The need** for detailed notes to guide the teacher in oral presentations of lessons was seen as greater for computer literacy and for probability and statistics (63%) than in other content areas, where the level of support was 50% to 60%. The PR and PT samples gave the idea strong support.

• Providing teachers with a syllabus that suggests topics and methods for each grade level, with specific times the topics should be introduced, received moderately strong support (71%) at the elementary level from the SP and TE samples. However, support for the same idea at the secondary level was minimal (57%).

• In-service materials to teach teachers the content of probability and statistics were supported by 85%, and in-service training on problem-solving methods was supported by 83%.

*11. Use of Audiovisual Aids*

• The use of films or videotapes received strong to moderately strong support from all samples (71% to 88%).

• Large-scale demonstration devices were rather strongly supported (usually by over 75%), especially by the SP, TE, and PR samples.

• The use of audiotapes fro drill and practice received support at the 65% level from all but the MA sample. However, tapes of lectures were negatively perceived by all samples.

*12. Logic, Deductive Methods, and Structure*

• Increasing the emphasis on curricula based on the logic of mathematics was given relatively little support (45%).

• There was esentially no support for increasing the emphasis on proof or formal axiomatic structures. For the latter, 32% favored decreasing emphasis and only 14% favored increasing emphasis.

• Developing logical thinking ability as a goal for whole numbers, geometry, and probability and statistics received very strong support (over 90%) from most professional samples. As a goal for computer literacy, support was only slightly less (80%). Lay samples also strongly supported the general goal (93%).

• Emphasizing reasoning techniques for ratio, proportion, and percent; fractions and decimals; and secondary school geometry received moderately strong support (61% to 82%). However, emphasizing logical reasoning principles in elementary school geometry was supported by only 23%.

• Symbolic logic as a topic for all students received very little support (31%), but minimal support (51%) was given to including symbolic logic for college-bound students who will not be science or mathematics majors.

• The goal of learning to read and interpret mathematical arguments was strongly supported (82%).

• The goal of teaching mathematics in order to impart an understanding of the structure of mathematics was strongly supported (82%) for whole numbers by professional samples and moderately supported (75%) as a generic item by the lay samples.

• The study of structural properties of number systems received higher support at the elementary level (69%) than at the secondary level (53%).

## MAJOR POINTS FROM THE PRIORITIES SURVEY

The priorities survey requested respondents to assign priorities for allocating funds, energy, or time to alternative practices.

*1. Development of New Materials at the Elementary Level*

• Problem solving received strong endorsement from *all* samples as the first priority for the development of new materials. (See Figure 22, where the data for problem solving and for whole numbers are given, and Figure 23, where the ranking for the five areas considered is presented.)

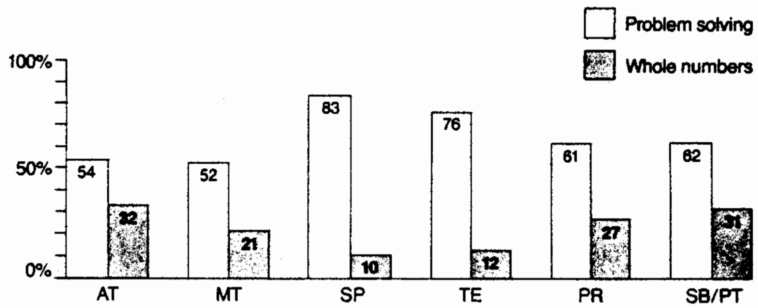



Figure 22. Percentages giving first priority to problem solving or whole numbers.

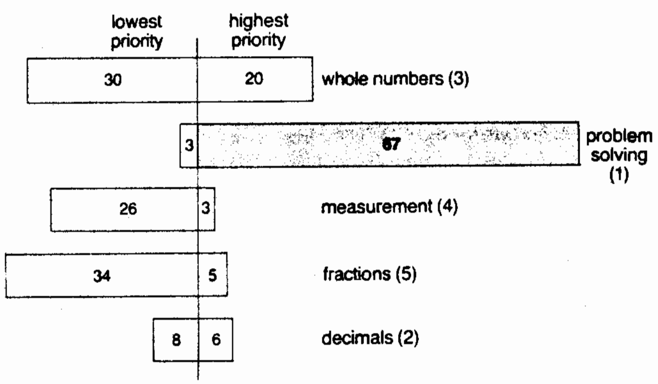



Figure 23. Percentages giving highest and lowest priority to five elementary areas.

• When asked why they rated problem solving highest, 56% indicated that they consider it an absolutely crucial skill for students, while 22% said that it is a major area of difficulty for teachers.

• Because rank was determined by averaging all responses, decimals was the area ranked second of the five for the development of new materials, even though few people gave it the highest priority (see Figure 23). Neither was it given lowest priority often, and this also contributed to its position. The AT, MT, SP, and TE samples gave more positive support to decimals than did other samples queried.

• Whole numbers was ranked third as an area for the development of new materials, with the highest priority coming from the AT and lay samples (see Figure 22). Ninety percent of those giving it highest priority did so because they consider it an absolutely crucial skill, while 61% of those who gave it lowest priority believe that adequate instructional materials already exist and 28% believe that the topic presents little difficulty for most teachers.

• Little support was given to the development of materials for measurement. Respondents indicated they believe that adequate materials already exist (32%) or that it is not as important for students to develop skills in this area (34%).

• Lowest priority was given to fractions; almost two-thirds (65%) said it was because the importance of fractions is diminishing, and 21% indicated that it is not as important as other skills.

*2. Use of an Additional Fifteen Minutes Each Day at the Elementary Level*

• Solving word problems attained first priority (by 31%) for the use of an additional fifteen minutes per day for mathematics in elementary schools (see Figure 24). All samples ranked it either first or second.

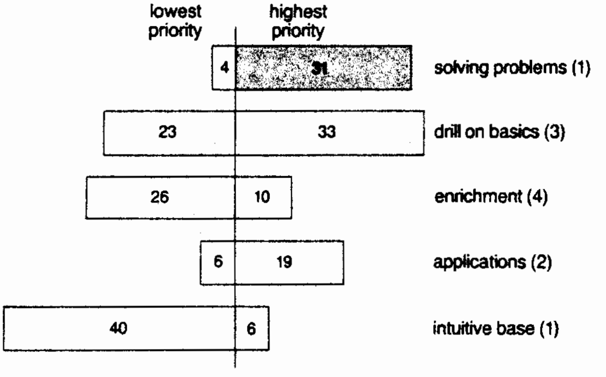



Figure 24. Percentages giving highest and lowest priority to the use of an additional fifteen minutes each day.

• While an even higher percentage (33%) gave drill on basics highest priority, many respondents also rated it lowest. Thus, the overall ranking for drill on basics was third. It was ranked very high by the MT and lay samples, while the SP and TE samples gave it much lower rankings. (The ranking of priorities was done by considering, statistically, the range of responses. Because of this, the percentage of support sometimes differs from the ranking; that is, one item may be ranked lower than another yet have a higher percentage selecting it as their first choice. An item's priority status was also affected when more ranked it low or very low.)

• Second priority went to studying applications of mathematics.

• Exploring enrichment topics was ranked fourth, while building an intuitive base for algebra and geometry was given lowest priority.

*3. Development of New Materials for Grades 7-12.*

• Computer literacy was identified as the area that should receive highest priority among the five choices for the development of new materials in grades 7-12 (see Figure 25). Most of the respondents (58%) who ranked computer literacy first indicated it was because they thought the importance of the area would increase during the 1980s.

• Algebra received second ranking for materials development, with those ranking it highest indicating most often (75%) that it was important for more students to develop skills in this area. The SB and PT samples were particularly strong in their support for algebra.

• Geometry was ranked third; of the 18% who gave it lowest priority, 47% believed that adequate materials for geometry already exist.

• Fourth priority was given to statistics, with the SP and TE samples ranking it much higher than the other samples. Eighty percent of these persons believed it is not as important to develop skills in this area as in other areas on the list.

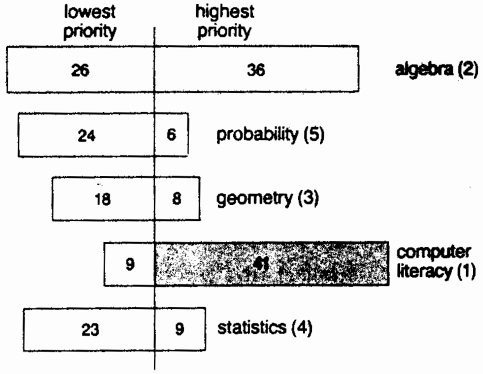



Figure 25. Percentages giving highest and lowest priority to five content areas for secondary school mathematics.

• Probability received the lowest priority for materials development. Eighty percent of those who ranked it lowest indicated that it is not as important for students to develop skills in this area as in other areas on the list.

*4. One Added Course at the Secondary Level*

• A course that helps students make decisions about buying and selling received highest priority among five suggested new (or extensively revised) courses for the high school curriculum (see Figure 26). Support for this course was weakest among the TE sample.

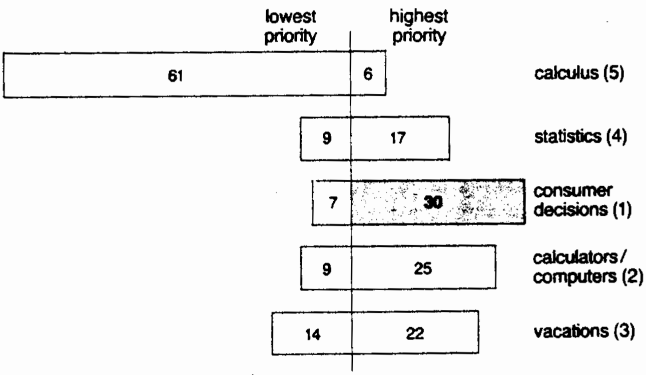



Figure 26. Percentages giving highest and lowest priority to five content areas for added course.

• A course that helps students understand how calculators and computers handle mathematics received second priority for being added to the curriculum. However, the SB and PT samples gave far less support to this choice than the other groups.

• All groups agreed that an additional calculus course should have lowest priority.

*5. Attention to Five Areas at the Secondary Level*

• Respondents clearly indicated that more attention should be given to applications of mathematics and to computer literacy (in that order) than to unified or interdisciplinary approaches or to structure in mathematics (see Figure 27).

• A unified approach received higher support from the SB and PT samples than from other samples.

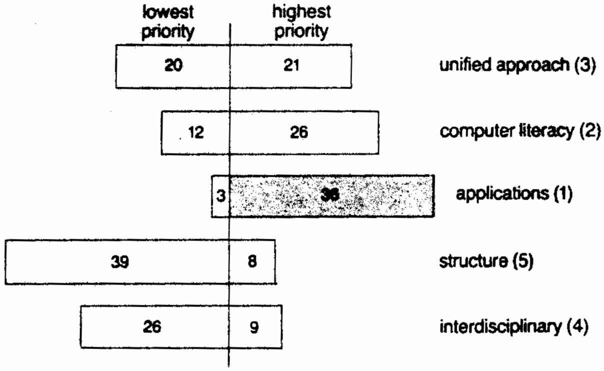



Figure 27. Percentages giving highest and lowest priority to five content areas.

### 6. *Attention to Five Additional Areas at the Secondary Level*

• Orienting mathematics to careers or vocations and to consumers was given higher priority than orienting mathematics to computers, college preparatory work, or recreational purposes (see Figure 28).

• The SP and TE samples gave a higher priority to computer orientation and a lower priority to vocational orientation than did other samples.

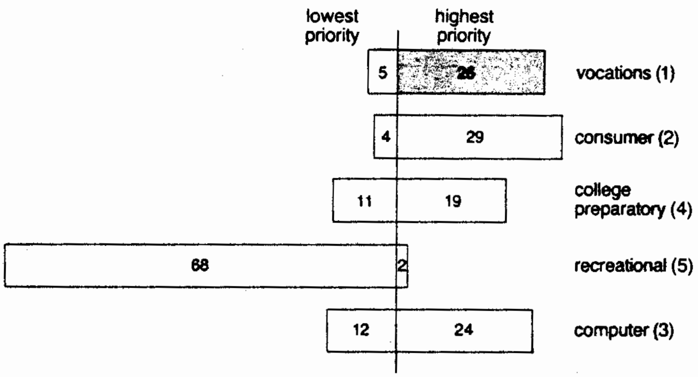



Figure 28. Percentages giving highest and lowest priority to five additional content areas.

### 7. *Types of Students*

• The needs of students with mathematics learning problems and other handicaps should have priority over the needs of four other types of students, according to all survey samples (see Figure 29). Many (45%) felt that this type of student has special needs that should be addressed through the curriculum.

• Second priority was given to the needs of inner-city or urban-area students by all samples. Most (61%) of those who gave highest priority to these students felt that they make up such a large proportion of the school population that significant resources should be devoted to meeting their needs.

• Female students received the lowest priority ranking of the five groups. Many respondents felt that these students had no special needs in mathematics. The SP and TE samples ranked this group higher than other samples did, however.

• Neither ethnic minorities nor students whose first language is not English received particularly high priority for special help in mathematics; they ranked third and fourth, respectively. Of those who gave ethnic minorities the lowest priority, 47% felt that these students as a group had no special needs in mathematics.

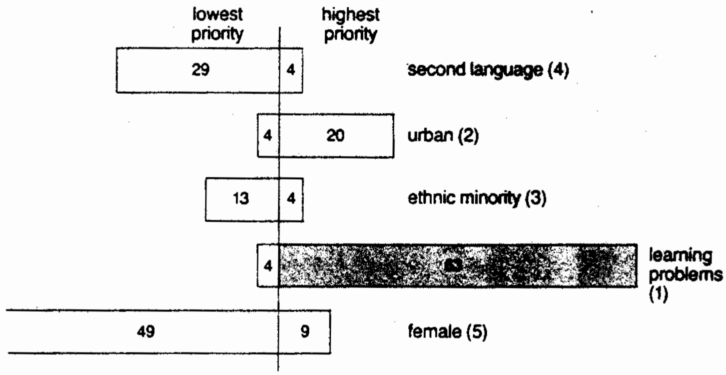



Figure 29. Percentages giving highest and lowest priority to five types of students.

*8. Teacher Education*

• When considering needs within teacher education, respondents gave methods first priority (except SB and PT, who ranked it second); and all samples gave materials last priority (see Figure 30).

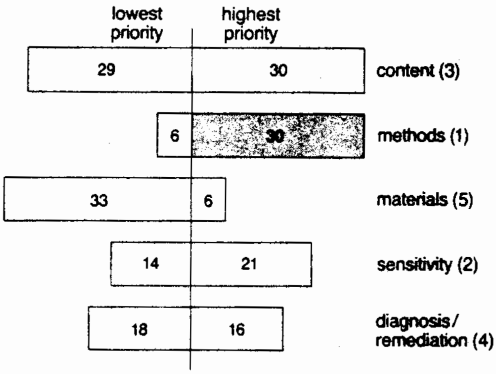



Figure 30. Percentage giving highest and lowest priority for needs in teacher education.

• Sensitivity to student needs was ranked second, with 21% giving it highest priority—less than for the third-ranked item. But ranking is based on the distribution of responses; only 14% gave sensitivity lowest priority, with most clustering at the middle level of support. The MT sample gave it less support than other samples, but it was clearly first choice for the SB and PT samples.

• Emphasizing mathematics content in teacher education was ranked third, receiving more support from the TE and MT samples than from others. The percentages ranking methods and content highest differed comparatively little (29.7% and 29.5%), but the percentages ranking each lowest differed markedly (methods, 5.8%; content, 28.6%).

• Diagnostic and remediation strategies was ranked fourth, with about the same percentages ranking it highest and lowest.

• The development of materials was given fifth priority by all samples.

*9. Across Areas*

• Respondents were next asked to assign priorities to each of the five broad areas addressed in the previous questions. Improved preservice and in-service teacher education was given top priority by the AT, MT, SP, and TE samples (see Figure 31), and 74% of those

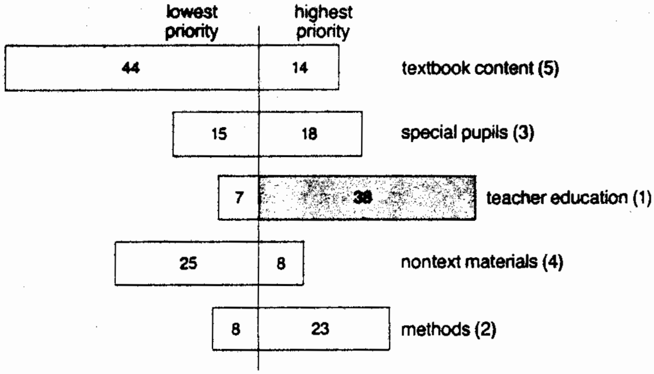



Figure 31. Percentages giving highest and lowest priority across areas.

who ranked it highest indicated they believed it would have far-reaching impact on mathematics education generally.

• Lay samples gave highest priority to the improvement of methods and techniques for teaching mathematics; professional samples placed this second.

• Developing special materials for students with special needs was ranked third. The many problems of these students and the lack of sufficient materials were cited by 37% and 33%, respectively, of those who ranked this highest, while the potential for less impact and the availability of sufficient materials were cited by those ranking it lowest (46% and 35%, respectively).

• The development of nontext materials was ranked fourth. Most who rated this lowest (42%) felt that it would have less impact on mathematics eduction, although many (35%) also felt that sufficient materials, methods, and understandings were already available.

• Lowest priority was given to improved mathematics content for textbooks, with many (66%) feeling that sufficient materials were already available.

*10. General Problems*

• Sixty-one percent felt that general problems facing teachers deserve priority over those problems specific to the teaching of mathematics. The PR sample was most certain of this (71%); the TE sample was least sure (48%).

• Problems not specific to mathematics classrooms that are of greatest concern were unmotivated students, reading difficulties, and classroom discipline, followed by lack of commitment to homework, lowering of academic standards, and irregular attendance.

• The problem of least concern to all samples was that of restrictions on instructional materials. Increased teacher workload, increased class size, the mixing of students with differing abilities, governmental restrictions, and decline in student ability were among the other problems ranked low.

• The lowering of academic standards and the lack of commitment to homework were of greater concern to the MT sample than to others.

• The SB and PT samples tended to be more concerned than the others about governmental restrictions (ranking it eighth) and less concerned about teacher workload (ranking it thirteenth.

*11. Distribution of Research Funds*

• There was clear agreement by all samples that first priority for the distribution of research funds should be given to studies of how children learn (42%).

• Second in ranking was teaching methods, with 19% giving it highest priority, while teacher education ranked third, although with a slightly higher percentage (24%) giving it highest priority. (More also gave it lowest priority, thus causing it to be ranked lower than teaching methods.)

• Research on varying types of materials (9%) and longitudinal assessment of achievement (6%) were ranked fourth and fifth.

*12. Methods of Attacking Problems*

• In-service education and preservice education were ranked highest as methods for attacking problems in mathematics education.

• Evaluation of learning, allocation of grants to local schools to improve their mathematics programs, and support of long-term research projects were ranked third, fourth, and fifth.

• National curriculum projects and grants to commercial firms were ranked lowest as methods for attacking problems in mathematics education.

• Although most people saw general problems of education as more critical than specific problems of mathematics education, support for research on general classroom problems ranked only twelfth of fifteen items.

• In general, funding for local projects was given priority over national projects.

*13. Accommodation of Talented or Gifted Students*

• If more mathematics were offered to talented or gifted students, all samples would give first choice to "a broad selection of enrichment topics" (60%), followed by work on computers and numerical analysis (23%). Third choice was topics in calculus and analysis (11%), while additional topics in algebra (4%) and geometry (2%) would be last choices.

*14. Comparison of Mathematics with Other Programs*

• Across groups, about the same percentages indicated that the mathematics program was "about the same" as other programs (46%) or "better" (43%). Very few indicated that the mathematics program was inferior (5%) (see Figure 32).

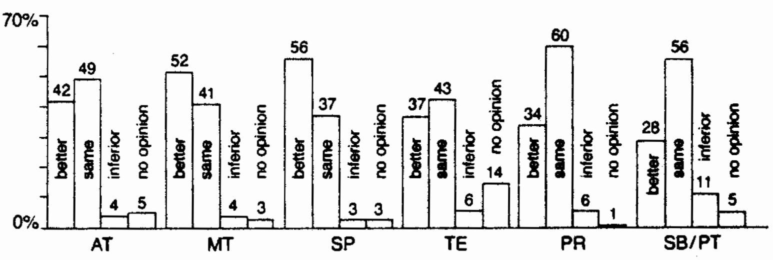



Figure 32. Results of comparison of mathematics with other programs.

*15. Need for Most Improvement*

• Mathematics for general education was seen as most in need of improvement by 61% of the respondents. This was followed by mathematics for the vocational student (24%). Only 14% saw mathematics for the college-bound student as most in need of improvement.

## PRISM DATA RELATED TO NCTM RECOMMENDATIONS

The eight recommendations in NCTM's *An Agenda for Action: Recommendations for School Mathematics of the 1980s* need to be considered in relation to the opinions reflected by the PRISM findings. Therefore, each is presented with a summary of the findings relating to it.

### Recommendation 1

*Problem solving must be the focus of school mathematics in the 1980s.*

• The PRISM data clearly indicate widespread support for problem solving in the school mathematics program.

• Problem solving was *consistently* ranked high in priority for increased emphasis in the 1980s.

• If a limited amount of money could be spent for the development of new materials, all samples would choose problem solving as having top priority for the expenditure of the funds.

• When asked to indicate how they would spend an additional fifteen minutes each day in elementary school mathematics, all samples gave problem solving either first or second priority.

• When the most accepted goals for problem solving are considered, it seems plausible to infer a reasonably high-level interpretation of problem solving—there is at least lip service to something more than routine exercises.

• Teaching a range of specific problem-solving strategies was also well supported.

• The idea was rejected that only problems that could be solved quickly should be given, leading to hope that there may be some acceptance of problem solving as a process in which students "question, experiment, estimate, explore, and suggest explanations."

• Providing in-service education to teachers on problem-solving methods was also strongly supported.

• There is consistent and strong support for increasing the emphasis on applications of mathematics throughout the curriculum.

• All samples strongly supported the importance of applications for solving problems in everyday life, gaining skills necessary for employment, making consumer decisions, and preserving student options in career or vocational choices.

Thus, the climate for implementing the first recommendation seems highly favorable. The task would seem to be less one of arguing for acceptance than one of hammering out the details; this will be no small task, for despite surface agreement, perceptions of what problem solving could entail in the school mathematics program differ widely. Nevertheless, the discussion would seem to hold great promise for working out a curricular change that would place the focus of the school mathematics program on problem solving.

**Recommendation 2**

*The concept of basic skills in mathematics must encompass more than computational facility.*

• The teacher and lay samples gave firm support to increasing the emphasis on basic skills in the 1980s; only the SP and TE samples were lukewarm to increasing this emphasis.

• When asked how they would spend an additional fifteen minutes each day in elementary school mathematics, the MT, PR, SB, and PT samples gave first priority to drill on basic number skills; the AT sample ranked this second to problem solving. It would seem that the teacher and lay samples still see a need to continue the back-to-basics movement into the 1980s, although the SP and TE samples ranked drill on basics quite low.

• The teacher samples tended to support spending over 50% of instructional time on drill and practice activities for whole numbers, fractions, and decimals, and the lay samples were even stronger in their support of this idea. Only the SP and TE samples disagreed with devoting so much time to drill and practice.

• Worksheets for providing practice at the end of each lesson were strongly supported by most samples.

• There was strong support by all samples for mastery of whole number computational skills before graduation from high school and only slightly less support for mastery of percentage problems as a condition for graduation.

• There was also support for many of the other points suggested in the *Agenda*'s recommended actions concerning this second recommendation. Problem solving, applications, estimation, measurement, geometry, and computer literacy, all endorsed by the NCTM (and all included on the National Council of Supervisors of Mathematics list of ten basic skills), received strong endorsement on items scattered throughout the PRISM surveys.

• Generally, however, the PRISM data do not show strong support for reduced emphasis on such "potentially obsolete" traditional skills as the long-division algorithm or computation with common fractions, which some persons argue are needed infrequently. In fact, the general posture (aside from the SP and TE samples) is conservative with respect to deleting topics from the curriculum. Nor is there strong support for including some "new" topics, such as probability and statistics. Calculator use was moderately supported, although seldom above the 50% level, but usually *after* pencil-and-paper algorithms had been taught. Only procedures made obviously obsolete by the calculator—for example, teaching the square root algorithm—were rejected by most samples.

• Support for teaching particular content in order to teach students to read mathematics received moderately strong support for several content strands. Having available materials with minimal reading requirements was given moderately strong support, but, in general, de-emphasizing reading in mathematics materials was given little support by any sample.

• Reading and interpreting mathematical arguments as a goal in geometry was strongly supported. Even stronger support was given to the goal of enabling students to read and think critically about graphs and data in other subject areas. And logical reasoning was supported as a goal of many content strands.

The chief difficulty in implementing the second recommendation may be agreeing on what encompasses basic skills and what level of attainment should be expected. If new topics or emphases are to be developed, then emphasis on, or inclusion of, others must decrease: these are the points where change will succeed or fail.

**Recommendation 3**

*Mathematics programs must take full advantage of the power of calculators and computers at all grade levels.*

• PRISM respondents indicated that they were well aware of the increasing emphasis that computers should receive in the mathematics curriculum. Surprisingly, lay samples gave even stronger support for increased emphasis on the use of computers than professional samples did.

• All groups queried indicated the desirability of having access to computers in mathematics classrooms at both elementary and secondary school levels. There was strong support for the development of new materials for computer literacy; all samples gave it high priority.

• Only half of the professional samples and less than half of the lay samples would require a computer literacy course for high school graduation. However, there was strong support by the professional samples for having all students receive *some* computer training before graduation, and the lay samples gave moderately strong support to the idea that at least one course in mathematics for college-bound students should make extensive use of the computer.

• Strong support was given to integrating computer literacy topics within the present curriculum.

• A course that helps students understand how calculators and computers handle mathematics was second in priority to a new course on consumer decisions.

• In a rank ordering of five areas for attention during the 1980s, computer literacy was second to applications. In a second rank ordering, computer orientation was third behind vocational and consumer orientations.

• Strong agreement was expressed by the professional samples for writing programs and for flowcharting, but only minimal support was given to writing programs as a requirement for high school graduation.

• Strongly supported for inclusion in the curriculum were the roles of computers in society, the types of problems computers can solve, and introducing alternative techniques for solving problems.

• Support for calculator use in schools was far more equivocal. About two-thirds indicated that students should have calculators; however, many samples tended to be restrictive about the use to be made of these calculators.

• Support was very low for using calculators instead of paper-and-pencil algorithms; rather, there was a strong belief by teacher and lay samples that their use should be postponed until after paper-and-pencil algorithms are learned.

• The SP and TE samples were decidedly more favorable toward increased use of calculators than other samples, and the PT samples were notably weaker in their support. But less than 40% of *any* sample indicated they should receive *less* emphasis.

• Requiring students who have not learned paper-and-pencil computation by the end of grade 8 to take a calculator course in grade 9 is acceptable to only half the respondents. In fact, using calculators with slower students was acceptable to only one-third of those sampled.

• Reactions to the use of calculators differed depending on the operation and the size of the numbers.

• Little support was given for using calculators when learning basic number facts or taking a test. Moderate support was given to using calculators for developing ideas and concepts with calculators, learning why an algorithm works, solving word problems, doing homework, solving equations, computing area, and making graphs.

• Strong support was given to using calculators for checking answers, doing a chain of calculations involving several different operations, and using trigonometry.

There may be sharper differences between groups on calculator issues than on any other issues in the PRISM surveys. It would appear that changing the curriculum to incorporate the use of computers could proceed much more smoothly than changing the curriculum to incorporate the use of calculators.

**Recommendation 4**

*Stringent standards of both effectiveness and efficiency must be applied to the teaching of mathematics.*

• This recommendation suggests the use of diverse instructional strategies, materials, and resources. The PRISM data clearly indicate that teachers want more, and more varied, teaching resources. A commitment to a wide variety of instructional strategies is less certain.

• Minimal support was given to increasing the emphasis on mathematics laboratories.

• Introducing ideas through laboratory investigations was similarly given minimal to moderate support, but strong support was given to providing booklets of experiments.

• Support was moderately strong for using manipulative materials, small-group instruction, and out-of-class activities.

• Increased emphasis on homework and text materials with daily homework problems were moderately to strongly supported.

The PRISM data do not clearly indicate which choices would be made, but a moderate level of acceptance was given to most reasonable alternatives. As stated for the second recommendation, the choices of what to teach (and what not to), and how to teach (and how not to), will need to be made clearly and decisively.

**Recommendation 5**

*The success of mathematics programs and student learning must be evaluated by a wider range of measures than conventional testing.*

• Few PRISM items addressed evaluation issues. However, evaluation of mathematics learning and achievement ranked third in priority of fifteen methods to attack problems in mathematics education, indicating it has rather high priority for the samples.

• Minimal support was given to increasing emphasis on minimal competency testing, and weak support was given to increasing the emphasis on norm-referenced testing. Substantial numbers indicated the emphasis should remain about the same.

• Minimal to moderate support was given to teaching whole number computation "to be able to do well on standardized tests."

**Recommendation 6**

*More mathematics study must be required for all students and a flexible curriculum with a greater range of options should be designed to accommodate the diverse needs of the student population.*

- Almost half of those in the lay samples (the only samples asked the question) responded that college-bound students should study four years of high school mathematics; 36% responded "3 years"; 13%, "2 years"; and only 3%, "1 year" or less.

- Forty-seven percent indicated that two years of high school mathematics should be required for high school graduation for *all* students, while 15% responded "4 years"; 25%, "3 years"; 12%, "1 year"; and 1%, "none." Thus it seems clear that there is support for more than a one-year requirement of mathematics.

- Minimal to moderate support was given to increasing the emphasis on individualization, but support was strong for having individual study materials available.

- Support was at least moderately strong for differentiating programs on the basis of career or vocational goals. Support levels were generally high any time a consumer-oriented or vocationally oriented approach to curriculum development was mentioned.

- There appeared to be resistance to requiring more of any one type of mathematics, beyond arithmetic, for *all* students: respondents would not require algebra, computer literacy, or probability and statistics of all students. But they would add computer literacy and consumer mathematics as *elective* courses, and they would expand on topics for college-bound students.

- There was moderate support for different algebra courses for students with different interests and abilities. However, requiring all students to take an algebra course as a requirement for graduation, or a historical or cultural mathematics course as a substitute for algebra, was rejected.

- Support was minimal to moderate for (a) including probability and statistics in the general mathematics or consumer mathematics course and (b) offering it as an elective after one year of algebra.

- Support was minimal for offering probability and statistics as (a) a senior-level advanced course for mathematics or science students of high ability, and (b) part of an interdisciplinary course.

- A full-year course in applied geometry was given moderate support as an elective, while a second year of advanced geometry in high school was supported and rejected by almost equal percentages.

- The needs of students with handicaps in mathematics learning and other handicaps were perceived as having highest priority among five groups, while urban students should have second priority. How to meet their needs was not, however, addressed. For instance, support for increasing the emphasis on materials for low achievers was only minimal to moderate.

- Developing special materials for girls was seen as having low priority or minimal support at best.

- Over three-fourths of the respondents believed that more emphasis should be given to gifted students, especially by offering a broad selection of topics.

It is apparent that there is support for a greater range of mathematics course options, as stated in the recommendation—but within the bounds of existing identified needs. Requiring a single type of mathematics course for all students is not supported; the inference is that additional options should be offered. However, if a wider array of geometry or probability and statistics topics is to be added to the curriculum, then clearly support must be developed.

**Recommendation 7**

*Mathematics teachers must demand of themselves and their colleagues a high level of professionalism.*

- A consistent message in the PRISM data is that improved preservice and in-service education of teachers—primarily on improved methods of teaching—is seen as the most promising resolution to problems of mathematics education.

- In-service education for teachers was given first priority in attacking mathematics education problems, followed by preservice education.
- Within teacher education, the top priority area to be addressed was methods, followed by work on sensitivity to student needs.
- At other points in the data, in-service education on, for instance, problem solving is given strong support.

Clearly, there is indication that teachers must develop and maintain teaching competence. This mandate for both preservice and in-service help is one that should aid in implementing the seventh recommendation.

### Recommendation 8

*Public support for mathematics instruction must be raised to a level commensurate with the importance of mathematical understanding to individuals and society.*

- Almost two-thirds of all respondents felt that general problems facing teachers deserve priority over those problems specific to the teaching of mathematics. Yet no sample ranked high the expenditure of research funds to attack general problems.
- Problems not specific to mathematics classrooms that are of greatest concern to mathematics teachers were unmotivated students, reading difficulties, classroom discipline, lack of commitment to homework, lowering of academic standards, and irregular attendance.
- Other areas, such as increased teacher workload, increased class size, mixing of students with different abilities, governmental restrictions, or decline in student abilities, were seen as being less serious.

It would appear that the constraints imposed by these general problems must be reduced if we are to deal effectively with those problems specific to mathematics teaching. Public support is obviously necessary in this undertaking, as noted by the eighth recommendation, just as increased support for education in general is needed.

## CONCLUDING COMMENT

Although there are many mathematical and psychological bases for making curriculum decisions, the final implementation of curricular change depends on the individual preferences of teachers, administrators, and parents at the local school level. The PRISM study provides a broad national view of attitudes and preferences and has implications for implementing all eight NCTM recommendations. There is also need to collect similar data from local communities.

The task proposed by the NCTM recommendations is large, but curricular change can be accomplished given the cooperative efforts of parents and other community members and teachers and other professionals. PRISM has provided some information on how these various groups perceive the mathematics curriculum and has indicated some differences and many similarities in beliefs across the samples. In essence, the data provide a general picture of the position of various groups as they relate to issues in mathematics education. This knowledge can help us as we proceed to develop plans for translating *An Agenda for Action* into practice.

Implementing the recommendations is not solely NCTM's task—it is a task for all concerned about the mathematics children learn in school.