

Conference on Basic Mathematical Skills and Learning
“The Euclid Conference”
Euclid, Ohio
October 4-6, 1975

Sponsored by the National Institute of Education (NIE)
U.S. Department of Health, Education, and Welfare

Volume I: Contributed Position Papers
Volume II: Working Group Reports

Direct quotes are denoted in italics

Why?

In the 1970s, many questions arose about the effectiveness of “new math” curricula developed during the previous two decades. Some mathematics educators and many parents believed that the topics in these mathematics curricula were too abstract and challenging for students and they questioned the lack of applications (Grouws & Cebulla, 2000). One reason for the perceived failure of the new math was inadequate implementation. “Teachers were not well equipped to deal with the demanding content of the New Math curricula” (Klein, 2003, p. 133). Standardized test results generally showed that students who were studying the new math were doing no better than students using other curricula.

As the National Science Foundation discontinued supporting new math programs, it began looking for solutions that would improve mathematics education across the country. Some felt it was time to go “back to basics” in mathematics. However, there was disagreement about the definition of basic skills among mathematics educators.

The National Institute of Education was formed in 1972 with four foci:

- *Help solve or alleviate the problems of, and achieve the objectives of, American education.*
- *Advance the practice of education as an art, science, and profession.*
- *Strengthen the scientific and technological foundations of education.*
- *Build an effective education research and development system.* (Volume II, p. 1)

In 1973, the NIE formed five priority areas to be researched, one of which was called the Essential Skills Program, later changed to the Learning Division of the Basic Skills Group. The purpose of this group was *to investigate, through research and development, ways to aid all children to obtain skills essential for functioning adequately in school and society* (Volume II, p. 1). This group initially focused on reading, but after one year extended its focus to include mathematics. From this new interest arose a fundamental question, *What are basic mathematical skills?* (Volume II, p. 1).

What?

At a May 1975 meeting of the NIE, the idea emerged for a conference on basic mathematical skills and two questions were identified for each of the participants to consider in a position paper. The two questions were:

- *What are basic mathematical skills and learning?*
- *What are the major problems related to children's acquisition of basic mathematical skills and learning, and what role should NIE play in addressing these problems?* (Volume II, p. 2)

Each invited participant to the Conference on Basic Mathematical Skills and Learning was asked to write a position paper answering these two questions.

The three-day conference was held in Euclid, Ohio. The first day, organized by James Wilson, included eleven presentations in which “status reports” were given regarding the various groups at the conference, including five NIE supported groups. Day two began with Gerald Rising summarizing the thirty-three position papers from the participants. Based on his recommendations, four working groups were formed around the following topics:

- *Classroom Instruction and Teacher Education*
- *Curriculum Development and Implementation*
- *Goals for Basic Mathematical Skills and Learning*
- *Research Priorities* (Volume II, p. 3)

The remaining time was spent in the four working groups.

Who?

Thirty-four invited participants, ten observers, and a group of NIE officials took part in the conference. Those people who were not involved in a topic group are listed here, while the other participants are noted with their topic groups.

Edward Begle	Stanford University
Paul Rosenbloom	Columbia University
Robert Baker	Southwest Regional Library
Thomas Butts	Case Western Reserve Laboratory
Edward Chalker	National Institute of Education
Carl Frederickson	National Institute of Education
John Mays	National Institute of Education
Kent Sullivan	National Institute of Education

Ed Begle did contribute a paper but was unable to attend the conference.

What was produced?

Two documents were produced from the Euclid Conference. Volume I included all of the contributed position papers. The published document contained the requested position papers that were edited and revised after the conference.

Volume II included all results and recommendations from the conference. Each of the four working groups published the ideas they generated from the conference discussion sessions. These four reports are summarized below. The participants are listed in each group. One asterisk (*) represents the steering committee members, two asterisks (**) represent the chairperson for the committee, and three asterisks (***) represent an observer to the group. The writer of the final report has his name in italics.

Report on Classroom Instruction and Teacher Evaluation

Charles Allen	Los Angeles Public Schools
Glenadine Gibb	University of Texas at Austin
Anna Graeber ***	Research for Better Schools, Inc
Joseph Harkin ***	Brockport State College
David Helms	Research for Better Schools, Inc
Leon Henkin **	University of California at Berkeley
Martin Johnson	University of Maryland at College park
John LeBlanc	Mathematics Education Development Center
Donald Ostberg ***	Northern Illinois University
<i>Gerald Rising</i> *	State University of New York at Buffalo
Joseph Rubinstein	Open Court Publishing Company

The participants determined that the main focus of mathematics education was to create a classroom environment where *students and teachers engage in the mutually supportive tasks of teaching and learning* (Volume II, p. 6). They emphasized the importance of improving classroom instruction to promote each student's thinking. They suggested three areas for NIE support:

1. Developing innovative teacher education programs that lay emphasis on clinical methods to better understand students' mathematical comprehension, on analysis of learning difficulties in mathematics, and on improvement of teaching techniques based on problem-solving.
2. Developing effective diagnostic tools and proper remediation materials. They believed that clinical centers should be established.
3. Developing instructional materials focused on problem solving and important mathematical ideas and procedures.

Report on Curriculum Development and Implementation

Aaron Buchanan	Southwest Regional Laboratory
James Fey **	University of Maryland at College Park and Conference Board of the Mathematical Sciences

<i>Peter Hilton</i> *	Battelle Research Institute and Case Western Reserve University
James Jordan ***	Washington State University
Burt Kaufman	CEMREL
Frédérique Papy ***	CEMREL
Dorothy Strong	Chicago Public Schools
Clifford Swartz	State University of New York at Stonybrook
Robert Wirtz	Curriculum Development Associates

This working group had two foci:

I. Basic Issues and Policy Guidelines

One focus was on *general policy or procedural guidelines applicable to all potential efforts in research and development* (Volume II, p. 10). The following recommendations resulted:

- A. An evaluation of the *behaviorist-humanist controversy* (Volume II, p. 10) and the repercussions it had on mathematics reform, including educational goals and assessment.
- B. A common implementation plan for new curricula so that studies could be replicated in multiple settings.
- C. The inclusion of teacher guides and professional development plans along with textbooks in all new curriculum projects.
- D. The creation and testing of all new curriculum projects in environments similar to those in which they were to be implemented.
- E. The involvement of members from all areas of mathematics education, as well as research-proven methods in the creation of all new curriculum projects.

II. Proposals for Research and Development

The second focus addressed research *related to specific aspects of the curriculum* (Volume II, p. 10), included the impact of calculators and computers, the integration of mathematics with its areas of application, implementation of curricula, creative teaching methods, alternative treatments of topics in arithmetic, and improved assessment materials.

The following statement contained within the proposals for research and development contradict earlier discussions on curriculum development: *Because this storehouse presently contains almost every conceivable type of curriculum that practitioners want, there is little justification for starting new and expensive curriculum projects* (Volume II, p. 13).

Also, while it was understood that a conflict existed between private curricula publishers and federally funded projects, the working group believed that the latter could spark great progress in the former.

Report on Goals for Basic Mathematical Skills and Learning

Jane Armstrong	National Assessment of Educational Progress
Peter Braunfeld	University of Illinois at Urbana

Earl Colborn	Byron, Frederickson & Colborn
Edward Esty *	National Institute of Education
Robert Hammond ***	Deerfield Academy
Norma Hernandez **	University of Texas at El Paso
Mary Koleski ***	Case Western Reserve University
Mitchell Lazarus ***	Educational Development Center
Dexter Magers ***	U.S. Office of Education
Ross Taylor *	Minneapolis Public Schools
Jerrold Zacharias	Education Development Center

The committee indicated that the goals they suggested might *initiate new directions in the learning of mathematics, and that “skill” [should] be interpreted in the widest possible sense, as a kind of shorthand for abilities, understanding, knowledge, and so on* (Volume II, p. 16).

The committee characterized three sets of goals that students should strive for throughout high school. *The goals presented here should not be considered as absolute* (Volume II, p. 16).

I. General Goals

General goals provided the meaning of understanding of mathematics, that is, they were the reason for studying mathematics. The committee considered general goals of teaching mathematics as: to assist children to be ready for life; to provide training for different productive occupations; and to help children in living a satisfying life. Mathematics education should help students improve the ability to think, to reason, and to make sense of situations that they face in their lives. Students should become self-confident about being able to operate effectively in a society where mathematical ideas are greatly used.

II. Basic Goals

Basic goals referred to the mathematics that was needed by adults in the society, such as the needs of consumers, citizens, workers, etc. Students should be provided with opportunities that would help them meet these basic goals before high school graduation. The committee offered the following set of basic goals:

A. Appropriate Computational Skills

It was suggested that the effect of the use of the calculator in the teaching of arithmetic be investigated. Since calculators were more available, people would not need to use long arithmetic procedures; the time that was spent on tedious drill and computation should be spent on important mathematical ideas and concepts that underlie the computation algorithms. Furthermore, students should be able to decide when to use calculators and when to develop algorithms. Students were expected to know basic single digit facts, the multiplication table, and to be fluent in simple computation problems.

B. Links Between Mathematical Ideas and Physical Situations

Students were to represent a real situation in mathematical terms, to analyze it using mathematical ideas, and to interpret it in order to reach conclusions about the original situation.

- C. Estimation and Approximation
Students were to make reasonable estimations by using simple methods, such as, rounding of numbers.
- D. Organization and Interpretation of Numerical Data, Including Using Graphs
Students were to create tables, charts and graphs when they are given a set of data and they should know how to read, analyze and interpret these same displays.
- E. Measurement, Including Selection of Relevant Attributes, Selection of Degree of Precision, Selection of Appropriate Instrument, Techniques of Using Measuring Instruments, and Techniques of Conversion Among Units Within a System
Students were to *measure length, distance, weight, volume, and temperature, and perhaps area and angles as well* (Volume II, p. 19).
- F. Alertness to Reasonableness of Results
Students should be aware of the fact that they can make mistakes sometimes, and they should check the rationality of their answers.
- G. Qualitative Understanding of and Drawing Inferences from Functions and Rates of Change
Students were to understand the concept of one quantity depending on another, and they should be able to use graphs and tables to determine the relationships among quantities.
- H. Notions of Probability
Students were to provide reasonable interpretations of predictions that are made based on probabilistic ideas.
- I. Computer Uses: Capabilities and Limitations (Gained through Direct Experience)
It was deemed necessary to have an understanding of what computers can and cannot do.
- J. Problem Solving
Problem solving was to be considered as a main goal that interrelates the general, basic, and further desirable goals.

III. Further Desirable Goals

These goals were written not only for gifted students, but also for those students who were more interested in mathematics than typical students.

- A. Recognition that Mathematics Is a Construct
Students were to recognize the internal considerations of the mathematics discipline.
- B. Ability to Reason Abstractly
Students were to construct and reason abstract arguments or proofs.

C. Enrichment of the Student's World

Studying mathematics should help students appreciate and understand events in an enhanced way.

D. Acquaintance with the Natural Notations of Mathematics

Students were to be familiar with internationally accepted mathematical notations.

E. Mathematical Modeling

Mathematical modeling was to be emphasized, because it helped students recognize certain aspects of situations and useful applications of mathematical ideas.

Report on Research Priorities

Nicholas Branca	Pennsylvania State University
Thomas Carpenter ***	Wisconsin Research and Development Center
Robert Davis *	University of Illinois at Urbana
Herbert Ginsburg	Cornell University
David Klahr	Carnegie-Mellon University
Mauritz Lindvall	Learning Research and Development Center
Eugene Nichols	Project for the Mathematical Development of Children
Joseph Payne	National Science Foundation
David Perkins	Harvard University
Thomas Romberg **	Wisconsin Research and Development Center
Israel Scheffler *	Harvard University
James Wilson *	University of Georgia at Athens

This working group focused on the development a set of research questions stemming from those posed by the conference participants.

These questions led to a list of recommendations for consideration by NIE in their plans for research:

Recommendation 1: *Increased support of research related to the learning and teaching of mathematics is critical.*

Recommendation 2: *Research efforts supported in mathematics education should be diverse in terms of breadth of problems attacked but should be concentrated and collaborative in terms of resources devoted to those problems selected for study.*

Recommendation 3: *It is imperative that researchers in mathematics education drastically enrich the procedures by which problems are conceptualized and explore a variety of methods, especially those proved successful in fields other than mathematics education.*

Recommendation 4: *Support for the collection and collation of information from prior work in mathematics education is essential.*

Recommendation 5: *Particular attention should be paid to problems of instrumentation and evaluation in mathematics education.*

Priority Recommendations

The following recommendations suggest foci for mathematics education research.

Recommendation 6: *Research in mathematics education should be supported to reflect a balance between investigations directed toward resolving questions of immediate practical urgency and investigations related to understanding learning and teaching.*

Recommendation 7: *Mathematics education research should be supported on questions that reflect all seven areas of investigation identified, but with preference to the first three areas.*

- A. *Identification and clarification of specific aspects of research problems*
- B. *The development of attitudes, concepts, skills, processes (focus on the individual child)*
- C. *Instruction (emphasis on the teacher and student-group interaction)*
- D. *School context*
- E. *Political and social context of the school*
- F. *Methodologies of research, development, and evaluation*
- G. *Teacher training*

Recommendation 8: *Mathematics education research should be supported on questions that reflect a balance among different levels of mathematical skills.*

- I. *Manipulation (questions regarding recall of facts, application of skills, algorithms, etc. and their routinization or their relationship to various aspects of mathematics)*
- II. *Quantitative and spatial comprehension (questions regarding how students quantify or model problem situations)*
- III. *Problem solving (questions regarding strategies and heuristics a student uses to solve problems)*

(Volume II, pp. 22-27)

The working group also provided specific research questions that aligned with the recommendations.

Thoughts on the State of Mathematics Education Today

The final section of the report contained a reflection on the conference from Peter Hilton and Gerald Rising. Here they elaborate on and offer solutions to the following ten problems:

1. *Problems of communication*

They saw this as *the central [problem] in curriculum development* (Volume II, p. 33) and suggested, *a new periodical designed to provide information about developments in mathematics curriculum and in pedagogy* (p. 34) to disseminate information about new curriculum projects.

2. *Science teaching*
They felt that collaboration needed to occur between math and science educators to determine the best approach to the teaching of scientific topics and mathematical applications to science.
3. *Schools in chaos* (mentioned in Wilson’s position paper)
Teachers today are subject to overwhelming pressure; they are expected to solve, or at least redress, all the problems of modern society and somehow, at the same time, to instruct the children in reading and mathematics. (Volume II, p. 35)
4. *Literature search*
Hilton and Rising believed that published research would provide few answers to the research questions posed and suggested that *recent clinical studies* be examined for their usefulness (Volume II, p. 36).
5. *The relationship of university academics to classroom teachers*
This was seen as *one of the most serious concerns that was repeatedly raised at the Euclid conference* (Volume II, p. 36). They felt there was a need for increase in reviewing grant proposals as well as an embracing of diversity and the differences it brought.
6. *Standardized tests*
We join our colleagues, Professor Jerrold Zacharias and Banesh Hoffman, in their campaign to suppress and supplant current models of standardized testing. We consider such testing to be not merely suspect, but actually seriously damaging to mathematics instruction. (Volume II, p. 37)
7. *Calculators*
Concern over teachers who put off calculator use until concept mastery had occurred and others who used them only for algebraic purposes was discussed. *We hope that the hand-held calculator will be used in the mathematics classroom predominantly to support and extend conceptual understanding of mathematics and to facilitate the application of arithmetical techniques to the solution of real life problems.* (Volume II, p. 39)
8. *False dichotomies*
Hilton and Rising felt that awareness should be made in “blending” positions instead of favoring one over the other due to the nature of views, which appear to be opposite, actually being “complimentary” (Volume II, p. 40).
9. *Problem solving*
The charge has been made against the new mathematics that it emphasizes too strongly the importance of mathematical structure, and that it overlooks the necessity to educate the child to be able to solve problems. (Volume II, pp. 40-41)
10. *The nature of mathematical usage today*
There needed to be a determination as to what mathematics was needed for the present and what math was to be needed in the future, including a study of the mathematical needs for skilled trades, technicians and applied sciences.

(Volume II, pp. 33-41)

Significance of the Report

“Back to the Basics” was a popular trend of the 1970s that gained much momentum and created “demands for programs and evaluations which emphasized narrowly defined skills” (NCSM Position Paper on Basic Mathematical Skills, 1977). This trend, which led to the Euclid Conference, was followed up by a discussion amongst the members of the National Council of Supervisors of Mathematics (NCSM). In January 1977, the NCSM published a position paper on basic mathematical skills (NCSM Position Paper on Basic Mathematical Skills, 1977).

The NCSM views basic mathematical skills as falling under ten vital areas. The ten skill areas are interrelated and many overlap with each other and with other disciplines. All are basic to pupils’ development of the ability to reason effectively in varied situations (NCSM Position Paper on Basic Mathematical Skills).

Ten Basic Skill Areas

1. *Problem Solving*
2. *Applying Mathematics to Everyday Situations*
3. *Alertness to the Reasonableness of Results*
4. *Estimation and Approximation*
5. *Appropriate Computational Skills*
6. *Geometry*
7. *Measurement*
8. *Reading, Interpreting, and Constructing Tables, Charts, and Graphs*
9. *Using Mathematics to Predict*
10. *Computer Literacy*

These ten areas mirrored the results of two Euclid Conference documents. This influence could be attributed to the fact that three members of the Euclid Conference, Dorothy Strong, B. Ross Taylor, and Edward Esty, were also members of the NCSM’s Task Force; Taylor and Esty were specifically involved with the third working group.

After both reports were published, there existed a false premise that a comprehensive list of basic mathematical skills could be established. Reflecting on the Euclid Conference publications and the NCSM’s Position on Basic Mathematical Skills, Reys and Kasten argued that any list needed to be flexible: *What constitutes basic mathematical skills is relative and is influenced by many factors, including the time and society in which one lives and one’s occupation, interests, and intellectual capabilities. Basic mathematics skills for some may not be basic for others. Basic skills are different today from what they were thirty years ago.* (Reys & Kasten, 1978, p. 109)

References

- Grouws, D. A., & Cebulla, K. J. (2000). Elementary and middle school mathematics at the crossroads. *American education: Yesterday, today, and tomorrow*. Chicago: The National Society for the Study of Education.

- Jones, P. S. (1970). *A history of mathematics education in the United States and Canada*. Reston, VA: National Council of Teachers of Mathematics.
- Klein, D. (2003). A brief history of American K-12 mathematics education in the 20th century. In J. Royer (Ed.), *Mathematical cognition*. Greenwich, CT: Information Age Publishing.
- National Committee on Mathematical Requirements. (1923). *The reorganization of mathematics in secondary education*. The Mathematical Association of America.
- National Council of Supervisors of Mathematics. (1977). *Position paper on basic mathematical skills*. NCSM.
- National Institute of Education. (1975). *Conference on Basic Mathematical Skills*. Vol. 1, Contributed Position Papers, Euclid, OH. Washington, DC: Author.
- National Institute of Education. (1975). *Conference on Basic Mathematical Skills*. Vol. 2, Working Group Reports, Euclid, OH. Washington, DC: Author.
- Reys, R., & Kasten, M. (1978). Changes needed in the current direction of minimal competency testing in mathematics, *The Mathematics Teacher*, 71(2). 108-112.