The Place of Mathematics in Secondary Education

The Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics 1940

Why?

During the 1920s and 1930s, the place of mathematics in the secondary schools was being questioned. Although enrollment at public schools increased in the early part of the twentieth century, the number of students taking mathematics courses was on the decline. Students were displaying a great dissatisfaction with the mathematics curriculum. There were a large number of students failing secondary mathematics, and the subject was taught with little insight into its everyday utility.

The 1923 Report by the National Committee on Mathematical Requirements focused on explaining the importance of mathematics in the school curriculum. Decrying the overuse of drill, calling instead for mathematical instruction promoting understanding, the mathematicians and mathematics educators who were the authors of the 1923 report sought to place mathematics in the schools on solid footing after the decline of the theory of mental discipline. Their efforts went for the most part unheard, however, due in no small part to the Kilpatrick Committee. W.H. Kilpatrick was the chairman of an NEA-sponsored committee that produced the 1920 report *The Problem of Mathematics in Secondary Education*. This report proposed changing the tradition of presenting mathematics in a logical ordering, insisting instead that topics in mathematics be selected only for their usefulness in other areas (NCTM, 1970, p. 193). Because the Kilpatrick Report undercut the efforts of the National Committee on Mathematical Requirements, mathematics continued to suffer from neglect throughout the 1930s.

What?

In response to this situation, the Joint Commission of the Mathematical Association of American (MAA) and the National Council of Teachers of Mathematics (NCTM) began working on this report in 1934. The commission questioned the feasibility of implementing the progressive education ideas as applied to mathematics, yet incorporated modern views on the way mathematics should be taught.

The Joint Commission received funding from the Progressive Education Association (PEA). The Commission met sparingly at the beginning of their work due to budgetary concerns. In 1937, the General Education Board provided the Commission $5000 (eventually increasing to $6500) to complete their work. The Commission developed the report, published in 1940, with subject and course specific recommendations for mathematics in grades 7-12. Because the members of the Commission believed that there is “no one perfect pattern of instruction in mathematics” (NCTM, 1940, p. x), it offered two different programs to accommodate a wide range of needs. In

this 253-page report, the Commission listed a set of guiding principles that would provide a vision for secondary mathematics and another set of principles to guide the sequence of topics included in the curriculum. The report also offered suggestions for dealing with lower-level and accelerated students.

Who?

Members of the Joint Commission

Representing the Association

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Representing the Council

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What was produced?

The Commission developed a ten-chapter report discussing the progression of mathematics education in the public schools, its current state of existence, and the Commission's vision for what should develop in the future for both students and teachers of mathematics. The report began with a review of the current status of mathematics due to the evolving demographics of the United States. The Commission felt that the changing times, based on industry and the technological movement, meant the needs of students were also changing and thus the mathematics curriculum must adapt as well. Their philosophy was guided by the belief that the laboratory approach for instruction allowed students to grasp mathematics at a higher level when compared to the method of drill and practice. The Commission also believed that mathematics "should have a prominent place in secondary education" (NCTM, 1940, p. 50).

The chapters are as follows:

Chapter 1: Looking at Modern Education and its General Aims
Chapter 2: General Objectives for Secondary Education
Chapter 3: The Place of Mathematics in Education
Chapter 4: The Mathematics Curriculum

Chapter 5: One Distribution and Organization of the Materials of Instruction, Grades 7-12
Chapter 6: A Second Curriculum
Chapter 7: The Problems of Retardation and Acceleration

The report outlined a "suggested grade placement chart" for grades 7-12 (NCTM, 1940, pp. 76-98). These suggestions provided specific topics to be taught at each grade level in the areas of arithmetic, geometry, graphic representation, algebra, and trigonometry. Appendix V of the report provides the grade placement chart for grades 7-12 in these areas of study. Because the Commission felt that one curriculum plan would not fit all of the needs for every school, the report also included a second curriculum plan which described changes that could be made to the first curriculum plan to meet the needs of other schools. Some of the changes in this alternate curriculum included developing a general math course that involved more extensive arithmetic for grade 9, developing a demonstrative geometry course for grade 10, devoting the entire year in grade 11 to the study of algebra, and offering a selection of courses for grade 12, including courses that would review previous courses.

Two of the more interesting chapters were Chapters 3 and 4. Chapter 3 answered many of the attacks on mathematics, while Chapter 4 gave specific guidelines for mathematics in the high school curriculum. These are described in more detail below. Chapter 3 was very uplifting to mathematicians and mathematics educators as it addressed many of the attacks on mathematics. Chapter 4 represented the first attempt at giving specific national curriculum standards.

Chapter 3—The Place of Mathematics in Education

Several important quotations from sections of Chapter 3 are included below to give a flavor of the Final Report. The sections were designed to answer the attacks on mathematics in a systematic and eloquent manner. The remarks at the end summarized the committee's feelings about the place of mathematics in secondary education. Within quotes, important ideas are highlighted in bold.

Mathematical Study as Training in Clear Thinking

"**Geometry** has been treated solely as geometry and not as a subject, which in addition to being a splendid example of deductive reasoning, important and interesting in itself, **can also serve the purpose of creating a critical attitude of mind toward deduction and thinking in general**" (NCTM, 1940, p. 39).

Mathematical Skills

"The doctrine of 'postponement,' like the doctrine of 'incidental learning,' however alluring to the shortsighted person and however valid in certain subjects, is indefensible in the case of mathematics. The subject is so extensive and so difficult, requiring systematic and protracted study, as to be unsuitable for the general application of either of these doctrines" (NCTM, 1940, p. 44).

Mathematics and Desirable Attitudes

"When he fails to understand some point, his teacher frequently can trace the difficulty back step by step to something the pupil believed he understood but which in reality he grasped only imperfectly. **The lesson to be learned from such an experience is much broader than the mathematics involved**" (NCTM, 1940, p. 47).

Remarks

"**The Commission believes that what has been said indicates that mathematics should have a prominent place in secondary education. There should be ample provision for courses beyond the ones that are required, conscientious efforts being made to influence pupils to continue mathematical study. Boys and girls… should be protected against becoming victims of the doctrine of incidental learning or the doctrine of postponement**" (NCTM, 1940, p. 50).

Chapter 4—The Mathematics Curriculum

Basic Considerations

The commission believed that since mathematics has a cumulative nature, a mathematics program should be formed as a connected sequence of units, that is, it should unify concepts of mathematics. The diversity of administrative divisions in the country was recognized, and it was suggested that creating guiding principles that might provide an explicit move toward educational harmony was necessary. The National Council Committee on Arithmetic was going to issue a separate report on elementary school mathematics; hence the commission did not suggest a program for K-6. However, they provided a list of goals that students should have achieved after completing the 6^{th} grade. The students should

- be familiar with basic concepts of arithmetic;
- understand place value;
- have a solid understanding of operations of whole numbers, integers, fractions, and decimals;
- know units of measurement;
- be able to recognize, name, and draw common geometric figures;
- use estimations and check the reasonableness of their answers.

A Tentative List of Guiding Principles

The Commission provided a list of guiding principles to create a transition towards educational harmony. They suggested that the basic elements of arithmetic, algebra, geometry, graphic representation, and trigonometry should be included in the curriculum. The Commission emphasized the importance of reaching every type of students, and they believed that while the foundations of mathematics is taught, the applications of the field should also be stressed by considering the capacity of each student. They recognized the need for modifications in the rate of progress and the level of comprehension for retarded students.

The commission recommended that each student's understanding of mathematics should be determined early in each year; students should realize that their every day work impacts their knowledge of mathematics because it is a cumulative subject; in order to allow students to acquire an understanding of concepts and principles, the definitions should be the product of the learning process; teaching should be based on insight rather than drill; and topics should be revisited in order to provide students opportunities to incorporate their new knowledge with their prior knowledge.

The commission also provided a list of principles describing the sequence of the topics that should be included in the curriculum. They stated, that the sequence should be "such that each topic will contribute definitely toward an ever-growing and more significant organization of the basic concepts, principles, skills, facts, relationships, types of appreciation, and fields of application, resulting in the development of a unified mathematical picture" (NCTM, 1940, p. 57). Problem solving and modes of thinking should be emphasized in the study.

Essentials of a General Program in Secondary Mathematics

The commission made recommendations in foundational fields of secondary mathematics for grades 7-12:

1. Number and Computation

Basic Concepts and Principles

Students should be able to
- name or identify concepts;
- give examples or informal explanations of the terms;
- develop formal definitions of terms (operations, numbers relations, etc.).

Fundamental Skills

Students should be able to
- use the four basic operations with integers, fractions, and decimals;
- use the measurement units in real life;
- read simple numerical tables.

2. Geometric Form and Shape Perception

Basic Concepts

Students should be able to
- name figures;
- sketch or draw figures to illustrate geometric terms;
- develop formal definitions of basic terms (parts of figures, geometric relationships, etc.).

Fundamental Skills

Students should be able to
- draw, measure, and construct common geometric figures;
- develop essential geometric relationships.

3. Graphic Representation

Basic Terms and Concepts

Students should be able to

- name or identify concepts;
- give examples or informal explanations of the terms;
- develop formal definitions of terms (slope, symmetry, etc.).

Fundamental Skills

Students should be able to

- construct a graph (with an appropriate scale and title) to represent a set of data from a table;
- interpret a given graph;
- (optional) draw a line that fits data that are approximately linear.

4. Elementary Analysis

Basic Concepts

Students should be able to

- name or identify concepts;
- give examples or informal explanations of the terms;
- develop formal definitions of terms (structural and functional terms, etc.).

Fundamental Principles

Students should be able to use the fundamental principles of algebra and elementary analysis in related applications.

5. Logical (or Straight) Thinking

Basic Terms and Concepts

Students should be able to

- understand the basic terms;
- recognize actual use of terms in real life (assumption or postulate, proposition, converse, conclusion, etc.).

Fundamental Principles

Students should be able to understand the assumptions and principles on which the structure of mathematics is based.

6. Relational Thinking

Basic Concepts

Students should be able to recognize, name and define terms (constant, variable, independent variable, dependent variable, one-to-one correspondence, function, formula, table, etc.)

Fundamental Skills and Abilities

Students should be able to

- read tables of values;
- calculate formulas for values of independent variables;
- interpolate in tables and graphs;
- construct formulas from contexts;
- determine the constants for formulas to approximately fit given data;
- recognize functional dependence.

7. Symbolic Representation and Thinking

Students should be able to translate quantitative statements into symbolic form and conversely, and appreciate the power of such translation.

Further recommendations of the report

The final four chapters of the report dealt with the issues of retardation and acceleration of students, mathematics in the junior college, the evaluation of the progress of pupils, and the education of mathematics teachers. The Commission suggested that cumulative records should be kept for each student to allow schools the ability to assign students to the appropriate section for their level. These records should include intelligence ratings and other facts from achievement tests in both reading and math, physical defects, health issues, economic status, plans for vocation and information on the parents. Because of the dramatic increase in the number of junior colleges in the U.S. (2 in 1902 to over 550 in 1940), the Commission devoted a chapter to the need for junior colleges to change their mathematics programs to meet the needs of the graduating students entering professions, including a commercial group, a vocational group and an academic group. With respect to teacher education, the Commission outlined specific programs for "(1) high school teacher of mathematics and a second subject, (2) high school teacher of mathematics alone, (3) junior college teachers" (pp. 200-203).

Significance of the Report

- The Joint Commission provided reasons for students to study mathematics that is beyond arithmetic.
- They suggested spiraling curriculum that addresses a variety of mathematics topics each year.
- The report outlined two complete mathematics curricula to be used in secondary schools.
- Cumulative folders were recommended for use to aid both teachers and schools in the proper placement of students.
- The report provided guidance to secondary schools and junior colleges with respect to mathematics instruction by suggesting specific pre-service teacher programs in mathematics regarding both content and pedagogy. These programs included:
    - A program for high school teachers of mathematics and a second subject;
    - A program for high school teachers of mathematics alone;
    - A program for junior college mathematics teachers.
- The report identified two curricula to be used at the junior college level: one for the "academic group" and one for the "semi-professional groups" (commercial and vocational groups).
- It recognized that retarded and accelerated students may need to be taught differently.
- Suggestions for improvement in the areas of assessment and evaluation were provided, including: the purposes of evaluation; limitations of current testing practices; advances of recent years; a program for the future.

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