

**The Reorganization of Mathematics in Secondary Education**  
**A Report by the National Committee on Mathematical Requirements**  
**under the auspices of**  
**The Mathematical Association of America, Inc.**  
**1923**

## **Why?**

Prior to the formation of this committee in 1916, the movement for reform in the teaching of mathematics had gained considerable headway in various parts of the country. However, the movement lacked coordination and unity. Many different societal voices were being heard: teachers, administrators, college presidents, and parents. Each was asking for improvements in the area of mathematics beyond the elementary level.

The needs of youth and the utility of mathematics had replaced mental discipline as major curriculum determiners. The failure rate was increasing, and the enrollment rate was decreasing in classical mathematics courses. There was an increasing concern for individual differences among students, and growing criticism of instruction that was dominated by drill and manipulative formalism. All of these things contributed to a concern for the quality of mathematics education by both mathematicians and professional educators. However, there was a conflict of opinions between these two groups regarding the problems of mathematics in secondary education relative to what should be taught, how much of it, to whom, how, and why. This committee was responding to a 1920 report from the NEA: *The Problem of Mathematics in Secondary Education*. The NEA committee was chaired by William Heard Kilpatrick and included no mathematicians on the committee, only educators. The Kilpatrick report recommended that a higher percentage of students needed less mathematics which caused the mathematics education community to defend their position that all students need more mathematics (NCTM, 1970).

## **What?**

In response to this state of affairs, the Mathematical Association of America (MAA) organized the National Committee on Mathematical Requirements (NCOMR) in the late summer of 1916. Their preliminary report was published in 1920 and the summary report in 1922 (NCTM, 1970). The committee created a number of reports which were collected into a 639-page volume named *The Reorganization of Mathematics in Secondary Education*, or the 1923 Report. The goal of the committee was to investigate the whole field of mathematics education from secondary school through college. They would then make recommendations on the best way to reorganize mathematics courses and improve mathematics teaching.

## Who?

The president of the MAA at the time, E. R. Hedrick appointed the original nine members of the committee. The members were:

### *Mathematicians*

David Eugene Smith, Teachers College, Columbia University  
J. W. Young, chairman, Dartmouth College  
A. R. Crathorne, University of Illinois  
C. N. Moore, University of Cincinnati (replaced Veblen)  
E. H. Moore, University of Chicago  
H. W. Tyler, Massachusetts Institute of Technology  
Oswald Veblen, Princeton University

### *Representatives of secondary mathematics teachers' associations*

Vevia Blair, Horace Mann School, New York City, Association of Teachers of Mathematics in the Middle States and Maryland  
G. W. Evans, Charlestown High School, Boston, MA, New England Association  
J. A. Foberg, vice chairman, State Department of Public Instruction, Harrisburg, PA, Central Association of Science and Mathematics Teachers, Crane Technical High School, Chicago

### *Added later*

A. C. Olney, Commissioner of Secondary Education for California  
Raleigh Schorling, The Lincoln School, New York City  
P. H. Underwood, Ball High School, Galveston, TX  
Eula A. Weeks, Cleveland High School, St. Louis, MO  
W. F. Downey, English High School, Boston, MA, Association of Teachers of Mathematics in New England (replaced Evans)

E. L. Thorndike, Columbia University, consented to act in an advisory capacity for the committee on questions related to psychology

## What was produced?

The committee produced a collection of reports in a volume entitled *The Reorganization of Mathematics in Secondary Education*. The document was published in two parts: Part II contained the studies and research conducted by committee members and other organizations, and Part I contained ideas and recommendations which were mainly the conclusions of the studies from part II.

The chapters were as follows.

## **Part I. General Principles and Recommendations**

Chapter 1: A brief outline of the report.

Chapter 2: Aims of mathematical instruction—general principles

### I. Practical aims

- a. Acquiring utility in performing important numerical computations
- b. Understanding the language of algebra
- c. Understanding and using elementary algebraic methods to solve problems
- d. Understanding and interpreting graphical representations
- e. Obtaining familiarity with common geometric forms and their properties

### II. Disciplinary aims

- a. Acquiring quantitative ideas or concepts commonly used in real-life situations
- b. Developing an ability to think clearly in terms of these ideas and concepts including analyzing, recognizing logical relations, and generalizing
- c. Acquiring positive attitudes and effective habits of mind
- d. Developing “functional thinking”—thinking in terms of relationships between variables

### III. Cultural aims

- a. Developing an appreciation of beauty in the geometrical forms found in nature, art, and industry
- b. Developing ideals of perfection regarding such aspects as logical structure and logical reasoning, for instance
- c. Developing an appreciation of the power of mathematics and its role in the development of civilization

### IV. The point of view governing instruction

- a. The main goal of mathematics teaching should be to develop powers of understanding and of analyzing relationships.
- b. Topics, processes, or drill techniques which do not promote the above powers should be eliminated.
- c. The mathematics curriculum should focus on “functional relation” as the central unifying theme.

### V. The organization of subject matter

- a. General courses which unify the topics of arithmetic, algebra, and geometry were recommended.
- b. Required courses for all students consisting of the most important mathematics for the student each year disregarding college entrance requirements for the grades seven, eight, and nine were recommended.
- c. The Junior High movement of 1920 was endorsed and the general adoption of this model was urged for the purpose of increasing efficiency of the teaching of mathematics.

### VI. The training of teachers

- a. Teachers should be given the best possible opportunity for professional preparation and improvement both before and during teaching.

- b. The report made a strong statement regarding mathematical instruction: *“It will be apparent from the study of this report that a successful teacher of mathematics must not only be highly trained in his subject and have a genuine enthusiasm for it but must have also peculiar attributes of personality and above all insight of a high order into the psychology of the learning process as related to the higher mental activities. Administrators should never lose sight of the fact that while mathematics if properly taught is one of the most important, interesting, and valuable subjects of the curriculum, it is also one of the most difficult to teach successfully”* (NCMR, 1923, p. 16).

### Chapter 3: Mathematics for years seven, eight, and nine

- I. Courses must contain the fundamental notions of arithmetic (including statistics), algebra, intuitive geometry, numerical trigonometry, and at least introduction to demonstrative geometry in a correlated/unified fashion.
- II. These courses should be required of all secondary school pupils.
- III. Teachers should include historical and biographical information throughout the mathematics courses to spark interest in the subject and give the impression that math is a growing subject.
- IV. A list of topics to be omitted or postponed was given, such as excessive algebraic drill, so that other optional topics, such as logarithms and financial applications, could be introduced earlier.
- V. Five possible plans for a junior high course sequence were given as examples of how the recommendations could be accomplished. Here is one of the plans; the other four plans were slight variations of this one.

#### Plan A

First year: Applications of arithmetic, particularly in areas as relate to the home, to thrift, and to the various school subjects; intuitive geometry.

Second year: Algebra; applied arithmetic, particularly in areas as relate to commercial, industrial and social needs.

Third year: Algebra, trigonometry, demonstrative geometry.

By this plan the demonstrative geometry is introduced in the third year, and arithmetic is practically completed in the second year. (Bidwell, 1970, p. 410)

- VI. The junior high was contrasted with the 8-4 system; if the 8-4 system is continued, the schedule of when arithmetic is taught needs to be adjusted to allow room for algebra and geometry.

### Chapter 4: Mathematics for years ten, eleven, and twelve

- I. All high schools should offer mathematics courses for years 10, 11, and 12, and encourage a large proportion of students to take them.
- II. Courses should prepare students for possible vocations and life in the real world.
- III. Recommended elective courses, open to students who have successfully completed the junior high requirements, include plane demonstrative geometry, algebra, solid

- geometry, trigonometry, elementary statistics, elementary calculus, history and biography.
- IV. Four different plans for a course sequence were given in the hope that they would be helpful, but none was meant to be superior to another.
  - V. Calculus was recommended for senior year in schools which have adequate teacher availability.

#### Chapter 5: College entrance requirements

- I. The purpose of college entrance requirements should be to assess the student's ability to benefit from college instruction.
- II. College entrance requirements in mathematics should be based primarily on the special knowledge and training required for the successful study of courses which the student will take in college.
- III. It is not desirable to separate prospective college students from other students in the early years of secondary school, therefore secondary mathematics courses should not be planned only to satisfy college entrance requirements.
- IV. As far as necessary mathematics instruction is concerned, there is no conflict between the needs of students who go to college and those who do not.
- V. Prominent college teachers ranked the value of topics as preparation for elementary college courses.
- VI. Definite recommendations were made to change college entrance requirements and the type of college entrance examination.

#### Chapter 6: List of propositions in plane and solid geometry

- I. The committee listed fundamental propositions that should form the common minimum of any standard geometry course. This list was a reduced version of the list given by the Committee of Fifteen (1911).
- II. The propositions were selected based on their usefulness to subsequent proofs and exercises, and their value in completing important pieces of theory.
- III. A distinction was made between propositions relating to plane vs. solid geometry.

#### Chapter 7: The function concept in secondary school mathematics

- I. Functions play a great part in the world around us; many things can be related by some functional relationship especially in the areas of algebra, geometry, and trigonometry.
- II. The idea of relationship or dependence between variable quantities should be conveyed to students using numerous concrete instances of such relationships.
- III. Functions should serve as a unifying element running throughout the instruction in the mathematics of the secondary school.
- IV. At the junior high level, function was viewed as relevant in work with formulas, graphing, and interpretation of data.

- V. If students learn mathematics through the unifying idea of function, algebra and geometry will have new meaning for them, and they will be able to better confront actual problems in their own lives.

#### Chapter 8: Terms and symbols in elementary mathematics

- I. Recommendations were given for words and symbols that should be used, and those that should not be used. For example, the committee recommended that the terms trapezium and rhomboid not be used, and by 1930 these terms had not appeared regularly in mathematics textbooks.
- II. Some recommendations were made for words that they would like to replace but for which they could not find an adequate replacement.
- III. An attempt was made to standardize mathematical writing.
- IV. There was also an attempt to simplify terms in elementary instruction.

### **Part II. Investigations Conducted for the Committee**

#### Chapter 9: The present status of disciplinary values in education by Vevia Blair

- I. A study of the experiments and discussions of psychologists on the subject of transfer was reported.
- II. Statements/logical inferences from these experiments and discussions were drafted.
- III. The opinions of forty leading educational psychologists with regard to the truth of these statements were solicited.
- IV. An analysis of the replies and an attempt to formulate conclusions from them was made.

#### Chapter 10: The theory of correlation applied to school grades by A. R. Crathorne

This chapter was an investigation into the correlation between school subjects.

#### Chapter 11: Mathematical curricula in foreign countries by J. C. Brown

This chapter was broken into five subchapters with the following titles:

- a. General arrangement of courses in typical schools of the various countries
- b. Summaries of the work in mathematics by years
- c. Graphic representation of work in mathematics
- d. Certain important points of difference between the work in mathematics abroad and in the United States
- e. Bibliography

#### Chapter 12: Experimental courses in mathematics by Raleigh Schorling

This chapter focused on experimental courses in the following schools:

- a. The Cass Technical High School, Detroit
- b. The University of Chicago High School, Chicago
- c. The High School of Commerce, New York City
- d. The English High School, Boston

- e. The Ethical Culture School, New York City
- f. The Horace Mann School for Girls, New York City
- g. The Lincoln School of the New Mexico Normal University, Las Vegas
- h. The Parker High School, Chicago
- i. The Stuyvesant High School, New York City
- j. The University High School of the University of Missouri, Columbia
- k. The University High School, Minneapolis
- l. The Trenton Junior High School, Trenton
- m. The Washington Junior High School, Rochester
- n. The Mary C. Wheeler School, Providence

#### Chapter 13: Standardized tests in mathematics for secondary schools by C. B. Upton

Sample tests were given in the following areas:

- a. Arithmetic
- b. Algebra
- c. Geometry
- d. Mathematical ability

#### Chapter 14: The training of teachers of mathematics by R. C. Archibald

A discussion concerning the training of teachers of mathematics was given for secondary schools in foreign countries and the United States. Also included was a list of accrediting agencies and a tentative standard for training and coursework.

#### Chapter 15: Certain questionnaire investigations

This chapter was broken into three small vignettes

- a. *Change of Mind Between High School and College as to Life Work*, by A. R. Crathorne
- b. *Mathematical Interests of High School Pupils*, by W. F. Downey
- c. *Importance of High School Mathematics as Indicated by Certain Questionnaires*, by Alfred Davis

#### Chapter 16: Bibliography on the teaching of mathematics by D. E. Smith and J. A. Foberg

Appendix: List of co-operating organizations

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### **Significance of the Report**

- Defined and defended the purpose of mathematics in secondary education against those who wanted to reduce its role in the curriculum

- Started a trend, during the remainder of 1920s and 1930s, for writers in yearbooks and the *Mathematics Teacher* to write many articles concerned with the aims of mathematics
- Designated a curriculum model for both junior high and high school, with the five models for junior high having the most impact
- Proposed an integrated course called General Mathematics FOR ALL, less dominated by arithmetic
- Recommended Calculus in high school
- Proposed other math electives such as “mathematics of investment,” “shop mathematics,” “surveying and navigation,” and “projective geometry”
- Proposed the function concept as unifier of algebra and geometry
- Initiated an organizational approach to curricular reform
- Stimulated the creation of new texts and methods, although implementation was minimal, and nontraditional material was usually placed at the end of the book
- Stimulated the evolution of Geometry texts with emphasis on thinking through original exercises, as opposed to memorization of book theorems
- Influenced the rejection of the theory of mental discipline in favor of ideas of transfer
- Caused college entrance requirements to be amended to include general tests to predict collegiate success and to move away from examining achievement for specific mathematics courses
- Caused the consideration of increased training of teachers to include general knowledge, professional knowledge, and specialized knowledge
- Influenced the increased involvement of mathematicians with school mathematics

But ultimately...

- No major change in practice occurred largely due to the depression of the 1930s and the inertia of maintaining traditional educational practices
- Over the next two decades, the views expressed in the Kilpatrick report exerted more influence than the 1923 Report (Klein, 2003)

Parts of the report were given to all superintendents in cities with populations greater than 2,500, all normal schools, 1,500 libraries, publishers of 300 newspapers and periodicals, and 4,500 individuals. The reaction to the NCMR report included much praise. “The reaction was most positive and constructive for those features pertaining to the junior high school curriculum” (NCTM, 1970, p. 208) in that new texts and method books were written to include the committee’s recommendations.

## Resources

Bidwell, J. K., & Clason, R. G. (1970). *Readings in the history of mathematics education*. Washington, DC: National Council of Teachers of Mathematics.

Kilpatrick, W. H. (1920). *The problem of mathematics in secondary education*. A report of the commission on the reorganization of secondary education, appointed by the National Education Association. *Bureau of Education Bulletin 1920, 1*, 1-24.

- Klein, D. (2003). A brief history of K-12 mathematics education in the 20<sup>th</sup> century. In J. Royer (Ed.), *Mathematical cognition*. Information Age Publishing.
- National Committee on Mathematical Requirements (NCRM). (1923). *The reorganization of mathematics in secondary education*. The Mathematical Association of America.
- National Committee on Mathematical Requirements (NCRM). (1927). *The reorganization of mathematics in secondary education* (Part I). A report by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America, Inc. Boston, MA: Houghton Mifflin.
- National Council of Teachers of Mathematics. (1970). *A history of mathematics education in the United States and Canada*. Reston, VA: National Council of Teachers of Mathematics.