Curriculum and Evaluation Standards for School Mathematics

National Council of Teachers of Mathematics
Commission on Standards for School Mathematics
1989

Why?

A number of key publications, including *The Underachieving Curriculum* (McKnight et al., 1987) and NCTM's *Agenda for Action* (1980) pointed at lagging student performance on national and international assessments and called for extensive changes in the way students were taught mathematics and in the mathematics they were taught. Recommendations about mathematics instruction were deeply influenced by the emergence of constructivism and technological advancements, while curricular recommendations were meant to help modern mathematics curricula reflect the increasing value placed on mathematical literacy and technological agility in an age of information.

To ensure the quality of mathematics programs of study, to outline a direction and goals for ensuing changes, and to help promote large-scale reform, NCTM created the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). In the words of the authors, the Standards were intended to be "statements of criteria for excellence in order to produce change" (p. 2).

What?

In 1986, NCTM established the Commission on Standards for School Mathematics and charged it with two tasks:

1. "Create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied to diverse fields."
2. "Create a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision." (p. 1)

Four working groups drafted and revised the standards during the summers of 1987 and 1988. Drafts were reviewed by the general membership of NCTM and in the end, the product was a document of consensus: a set of 54 standards for the composition of mathematics curricula as well as the evaluation thereof. "In it a vision is given of what the mathematics curriculum should include in terms of content priority and emphasis" (p. v).

Who?

NCTM Commission on Standards for School Mathematics

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Italics denote the individual who chaired the working group.

What was produced?

The commission outlined the following as new goals for society:

1. Mathematically literate workers
2. Lifelong learning
3. Opportunity for all
4. Informed electorate (p. 3).

The commission outlined the following as new goals for students:

1. Learn to value mathematics
2. Become confident in their ability to do mathematics
3. Become mathematical problem solvers
4. Learn to communicate mathematically
5. Learn to reason mathematically (p. 5).

The standards are organized by grade band, with separate sections for K-4, 5-8, and 9-12. In each section, the Commission first included three standards for Problem Solving, Communication, and Reasoning that reflect the different expectations for students and instruction at each level. The fourth standard, Mathematical Connections, serves to emphasize that mathematics be taught cohesively and that connections between procedures and ideas be made.

Three assumptions were made about mathematics that provided a framework for the Curriculum Standards and shaped the treatment the commission gave to each standard. These assumptions of mathematics were imbedded in the Standards:

1. Learning mathematics is an active process. "Knowing" mathematics means "doing" mathematics.
2. Mathematics is relevant to more fields and requires quantitative knowledge not found in traditional sequences of algebra, geometry, precalculus, and calculus.
3. Technology has eased the burden of calculation and representation, and changed the nature of mathematical problems and methods of solution.

Assumption 1: Learning is an active process.

Drawing from a growing research base in psychology, the Commission claims a constructivist view of learning. Recommendations for classroom activities and instruction include a variety of forms including "appropriate project work, group and individual assignments, discussion between teacher and students and among students, practice on mathematical methods, and exposition by the teacher" (p. 10).

The Commission specified expected student activities associated with doing mathematics. These specifications were made in conjunction with each standard. Two general principles guided the description of student activities associated with doing mathematics:

1. Activities should grow out of problem situations;
2. Learning occurs through active as well as passive involvement with mathematics. "Instead of the expectation that skill in computation should precede word problems, experience with problems helps develop the ability to compute" (p. 9).

Assumption 2: Mathematics has changed.

Curriculum must provide opportunities to develop an understanding of mathematical models, structures, and simulations applicable to many disciplines, not just engineering and physical sciences. These fields include business, economics, linguistics, biology, medicine, and sociology.

Focus and discussion sections relevant to each standard discuss the purpose and reasoning behind the inclusion of each standard. Further, methods of implementing specific standards in classrooms are presented and discussed. It is in these discussion sections that student activities are presented. This section was "meant to convey the spirit of this vision about both mathematical content and instruction" (p. 10).

Assumption 3: Changes in technology have changed the nature of problems and the methods used to investigate them.

Recommendations were made regarding the availability and use of technology in the classroom. Calculators and computers are tools available to help simplify problems for students, but technological training is not the end goal. The vision of the Commission was that students would learn the fundamental mathematics that students will need and that technology would assist in that learning.

The following recommendations were made regarding availability of technology:

1. Appropriate calculators should be available to all students at all times.
2. A computer should be available in every classroom for demonstration purposes.
3. Every student should have access to a computer for individual and group work.
4. Students should learn to use the computer for processing information and performing calculations to investigate and solve problems (p. 8).

It was the intent of the authors that these standards should be implemented in such a way that each child is reached. Implementation of these standards should take into account the unique characteristics of students in each grade band. No child should be denied access to the study of one topic because of failure to master another.

Curriculum Standards for Grades K-4

Need and Direction for Change

The K-4 Working Group stated that the present K-4 mathematics curriculum "is narrow in scope; fails to foster mathematical insight, reasoning and problem solving; and emphasizes the rote activities" (p. 15). Students were receiving rules and procedures as passive learners, rather than making sense of mathematics as active learners.

A curriculum that reflects the new goals for students should be developmentally appropriate. It should pay attention to what young children bring to their study of mathematics benefiting from their experiences and intuitions, and capitalizing on their natural learning patterns. The curriculum should address the significance of the qualitative dimensions of children's thinking, focusing more on understandings than on skills. It should "build beliefs about what mathematics is, about what it means to know and do mathematics, and about children's view of themselves as mathematics learners" (p. 17).

Underlying Assumptions for Instruction and Mathematics

The early education Standards were shaped by considering the following basic assumptions about the K-4 mathematics curriculum. It should:

- be conceptually oriented, emphasizing mathematical concepts and understanding by allowing students to construct the meaning of mathematics;
- allow students to get involved in doing mathematics by encouraging them to explore, develop, and discuss mathematical ideas;
- highlight the importance of improving students' thinking and reasoning abilities, developing confidence about "their ability to think and communicate mathematically, to solve problems, to make appropriate decisions in selecting strategies and techniques, to recognize familiar mathematical structures in unfamiliar settings, to detect patterns, and to analyze data" (p. 18);
- recognize the significance of the application of mathematics, giving students a sense that mathematics is a subject that is applied to real life;

- include a variety of content areas such as measurement, geometry, statistics, probability, and algebra, in addition to arithmetic;
- require appropriate use of calculators and computers, encouraging students to explore mathematical ideas in efficient ways.

These standards should be implemented in such a way that each child is reached. No child should be denied access to the study of one topic because of failure to master another.

Summary of Changes in Content and Emphasis

In grades K-4, NCTM recommended changes to the content and emphasis of the mathematics being taught. Decreased attention was to be placed on the rote use of symbols and operations, and increase attention placed on number sense, estimation, and reasoning. Teaching thinking skills for basic facts, mental computation, and the use of calculators for complex calculation replaced tedious practice with pencil-and-paper algorithms and rote memorization of basic facts. Increased attention was to be placed on geometry and measurement, extending beyond the naming of geometric figures and converting between units of measure. New topics would include the properties of geometric figures and relationships between them, developing spatial sense, concepts related to measurement, and estimation of measure.

Recommendation of new topics to the elementary curriculum included probability and statistics, patterns and relationships, and a change in the definition of problem solving. Decreased attention was to be placed on teaching students to use clue words to determine appropriate operations, and problem solving was to become solving word problems with a variety of structures, using everyday problems, applying mathematics, studying patterns and relationships, and teaching problem-solving strategies.

Changes recommended for instructional practices were to take a problem-solving approach to instruction and to include more use of manipulative materials and technology, cooperative work, discussion of and writing about mathematics, questioning, justification of thinking, and integration of content. Decreased attention was to be spent on rote practice and memorization of rules, problems with one answer and one method, written practice, and teaching by telling.

Curriculum Standards for Grades 5-8

Need and Direction for Change

Mathematics is an area of study that should be appreciated by all students in grades 5-8. Unfortunately, because of the current emphasis on computational facility, many students in these grade levels find the subject arid and dull. This computational approach has failed to tap into the beauty of mathematics and the characteristics of the students in this grade range. The current curriculum revolves around rehashing the same topics grade after grade; new and important ideas occur in the final chapters of the book and are often neglected for lack of time. Furthermore, the existing curriculum in some schools requires the mastery of elementary arithmetic before students can move on to a broader curriculum. This gateway approach to middle school mathematics denies some students access to higher mathematics.

An ideal curriculum should enhance student knowledge in many areas. Computational proficiency should not be the only basis for school mathematics—basic material should include algebra, geometry, probability and statistics. Technology greatly facilitates student entry into these topics, including especially students who have heretofore not exhibited proficiency with pencil-and-paper computation. The additional exposure to these additional topics will introduce novel uses for and additional practice with computations taught prior.

Underlying Assumptions for Instruction and Mathematics

As this focus shifts, the mathematics curriculum should provide students with new problem solving opportunities that renew motivation for learning and provide context for the mathematical skills they are learning. Finding real-life applications for concepts and skills gives students an additional reference to remember skills they have forgotten. This increases their ability to solve future problems independently. The context provided also gives students concrete language to communicate their ideas to one another and with the teacher.

Instruction should include allowing students to wrestle with problems that are not well-defined. They should experience problem formulation and the generation of additional questions for exploration. Activities and explorations of problems should engage students both intellectually and physically. "Middle grade students are especially responsive to hands-on activities in tactile, auditory, and visual instructional modes" (p. 67).

The availability of the following materials is assumed:

- Manipulative materials and tools such as spinners, tiles, cubes, geoboards, pattern blocks, compasses, rulers, protractors, scales, scissors, graph paper, grid and dot paper.
- Access to appropriate resource materials with which to generate interesting and relevant ideas for exploration.
- Technology consistent with the recommendations previously mentioned. Calculators should be appropriate and include functions applicable to the tasks included envisioned for the curriculum.

Summary of Changes in Content and Emphasis

In grades 5-8, NCTM recommended changes in the content and emphasis of the mathematics being taught. Students, expected to be more active in their learning, should be asked to pursue open-ended problems and extended problem-solving projects where they investigate and formulate questions from problem situations. Students should be asked to use multiple representations for these situations including verbal, graphical, geometrical, and symbolic. Diminished attention should be placed on practicing routine, one-step problems and those problems that are categorized by types.

Students should discuss, write, read, and listen to mathematical ideas, rather than provide short answers, or purely numerical solutions. More attention should be spent on reasoning, placing the student in the role of mathematical authority rather than the teacher. The connections between topics and disciplines and applications of mathematics should be emphasized and skills should no longer be developed out of context or in isolation.

As in grades K-4, continuing emphasis should be placed on developing number and operation sense and creating, rather than memorizing and following, algorithms and procedures. Estimation and developing a sense of reasonableness in solution is important, as is exploring the relationships among the different representations of fraction, integers, decimals, and rational numbers. During these grades, students should develop an understanding of ratio, proportion, and percent.

Additional topics to be added to the 5-8 curriculum include patterns and functions, statistics, probability, and geometry. In many cases, these topics existed only as facts for students to memorize. Instead, students should engage in these topics in order to model, describe, analyze, evaluate, and make decisions about problem situations. Students should learn to estimate with and use measurement to solve problems rather than memorize conversions within and between measurement systems.

Curriculum Standards for Grades 9-12

Need and Direction for Change

Historically, the goals of secondary school mathematics have been to create productive citizens who are ready to pursue additional study at post-secondary institutions, or are ready to enter the workforce with no additional training. These goals have not changed; however, the very nature of that citizenry has. High school graduates are no longer likely to remain in a chosen career field for their entire lives and must be prepared for a variety of occupational experiences. The occupational experiences of the Information Age will differ from those of the Industrial Age, requiring all high school graduates, not just those entering post-secondary education, to be mathematically literate.

A school mathematics curriculum should be organized to maximize access of students to mathematics. In order to meet the needs of a variety of different students and achievement levels, it is the recommendation that students be provided with enrichment lessons when appropriate, rather than removing content for those students who have not demonstrated high ability or interest. In this way, students with exceptional talent should not be accelerated through their study by altering the content of the proposed standards or by stripping them of depth and character, and students who lack computational facility are not denied the opportunity to study the core curriculum.

Underlying Assumptions for Instruction and Mathematics

Underlying the pursuit of these goals are the following assumptions:

- Students entering grade 9 will have experienced mathematics in the context of the broad, rich curriculum outlined in the K-8 standards and the level of computational proficiency suggested therein will be expected of all students.
- Although arithmetic computation will not be a direct object of study in grades 9-12, number and operation sense, estimation skills, and the ability to judge the reasonableness of results will be strengthened in

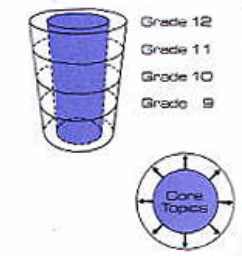

Fig. 1. A differentiated core curriculum

the context of applications and problem solving, including those situations dealing with issues of scientific computation.
- Scientific calculators with graphing capabilities and a computer for demonstration purposes will be available to all students at all times and students will have access to computers for individual and group work.
- At least three years of secondary mathematical study will be required of all secondary school students, and four years will be required for college-intending students.
- These four years of mathematical study will revolve around a broadened curriculum that includes extensions of the core topics and for which calculus is no longer viewed as *the* capstone experience.
- All students will study appropriate mathematics during their senior year.

Summary of Changes in Content and Emphasis

The shift from memorization of facts and procedures and computational proficiency toward conceptual understanding, multiple representations and connections, mathematical modeling, and problem solving continues in the recommendations for grades 9-12. Traditional topics retain their stature, but change in focus. Content should be presented in an integrated fashion—emphasizing the connections between topics such as algebra and geometry. Graphing utilities play a major role in helping students understand the connections. In addition to algebra, geometry, trigonometry, and functions, students should encounter topics from statistics, probability, and discrete mathematics.

Variety is recommended in the instructional methods used in classrooms in order to "cultivate students' abilities to investigate, to make sense of, and to construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems" (p. 125).

Evaluation Standards

The evaluation standards consist of 14 standards, categorized by focus: General Assessment, Student Assessment, and Program Evaluation. The main purpose of evaluation is to help teachers better understand what students know and make meaningful instructional decisions. The focus of evaluation is on what happens in the classroom as students and teachers interact. These standards call for changes beyond the mere modification of tests. Instead, they propose that assessment be an integral part of instruction, using multiple means of gathering information. Instruction and curriculum should be considered equally in program evaluation.

Significance of the Report

1. Projects

   As a result of the 1989 *Standards,* the National Science Foundation funded the creation of the following projects:

Elementary

- Everyday Mathematics, K-6 (UCSMP)
- Investigations in Number, Data, and Space, K-5 (TERC)
- Math Trailblazers, K-5 (TIMS)

Middle Grades

- Connected Mathematics (Michigan State University Connected Math Project (CMP)
- Mathematics in Context (Wisconsin Center for Education Research)
- MathScape: Seeing and Thinking Mathematically (Education Development Center)
- MATHThematics (STEM) (University of Montana)
- Pathways to Algebra and Geometry (MMAP)

High School

- Contemporary Mathematics in Context (Core-Plus Mathematics Project)
- Interactive Mathematics Program (IMP)
- MATH Connections: A Secondary Mathematics Core Curriculum (CBIA)
- Mathematics: Modeling Our World (COMAP)
- SIMMS Integrate Mathematics (SIMMS)

2. A dramatic increase in the use of technology, but especially handheld graphing calculators at the high school.
3. Dynamic geometry and statistical software was developed in support of student investigation of mathematical ideas.
4. Instructional philosophies and practices of some mathematics teachers began to shift to a more constructivist approach.
5. Contemporary applications of mathematics became available for study at all levels.
6. A strong need for mathematics content and pedagogical professional development for teachers became apparent.

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