

Running Head: Video-Case Curriculum

Using a Video-Case Curriculum to Develop Preservice Teachers' Knowledge and Skills

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Although the use of cases in preservice teacher education has become increasingly common, the majority of the cases available commercially—both written and video—were developed for use with inservice teachers. We describe our adaptation of one such video-case curriculum—*Learning and Teaching Linear Functions: Video Cases for Mathematics Professional Development, 6-10* [LTLF] (Seago, Mumme, & Branca, 2004)—for use with preservice teachers in a middle school mathematics methods course, and illustrate the nature of the growth in the preservice teachers' mathematical knowledge for teaching and in their development of a reflective stance towards teaching that occurred as a result of their engagement with these materials.

The LTLF Video-Case Curriculum

The LTLF video-case professional development curriculum was developed under a grant from the National Science Foundation to provide professional development for teachers in grades 6-10. The curriculum has a dual focus on developing teachers' understanding of the mathematics of linear functions and their understanding of pedagogical issues associated with teaching this topic. It includes a foundation module, consisting of eight three-hour sessions, and four extension modules, each consisting of two or three three-hour sessions. Extensive facilitation resources accompany each module. As a general sequence of activities, participants individually engage in mathematics tasks as learners and discuss the mathematics as a group before they watch students engaged with the same tasks via one or two short video clips. After a video clip is viewed, participants are given time to reflect on the video individually, recording mathematically and pedagogically interesting teaching moments in a journal and using the provided video transcript as a source of evidence to

support their observations. They then participate in a whole-group discussion in which participants share the teaching moments they identified and engage with the ideas that others put forth.

Rationale

The middle school mathematics methods course in which the LTLF curriculum was used is the first in a series of three methods courses that Western Michigan University's mathematics department offers for prospective secondary school teachers. This course is required for both mathematics education majors and minors, and is typically taken during students' sophomore or junior year. The LTLF curriculum was used for the first six weeks of the methods course. During the remaining eight weeks of the semester, the preservice teachers participated in group field experiences in local middle school mathematics classrooms, examined and compared middle school mathematics curricular materials, and considered state and national standards and benchmarks for middle school mathematics. For additional details, see the LTLF preservice addenda materials (Mumme & Seago, in development). We incorporated the LTLF curriculum into the course for two important reasons: (1) its assumptions and goals were compatible with those of the course; and (2) it provided a coherent set of curricular materials. These reasons are explored in more detail below.

Compatible Assumptions and Goals

The main focus of this NCTM *Standards*-based course is on teaching for student understanding by accessing and building on student thinking. We start with the assumptions that teaching mathematics is complex and requires a deep understanding of mathematics, and that inquiry and analysis are necessary to improve practice—assumptions that guided the

development of the LTLF materials. Reflecting these assumptions, the video cases of classroom teaching are intended to stimulate inquiry and reflection, and are not presented as exemplars of teaching. We want the preservice teachers in the course to begin to develop the skills and dispositions needed to teach for student understanding, including deepening their understanding of middle school mathematics. Because of its focus on both mathematics and pedagogy, and the opportunities it provides for focusing on student thinking, the LTLF curriculum is an ideal fit for the course.

Coherent Curriculum

Prior to our adoption of the LTLF curriculum, the cases of teaching used in the middle school methods course were drawn from a variety of sources. Although we kept the focus of each case on student thinking, it was difficult to weave the cases together to build a coherent, integrated curriculum. By design, the videos in the LTLF curriculum build on each other mathematically, providing opportunities to make connections among the student thinking in various video clips. An important part of our adoption was the decision to use the entire LTLF foundation module intact, rather than select a small number of video cases to use periodically in the course. While this decision resulted in the loss of some content that previously had been part of the course—most noticeably an extended treatment of rational numbers—we felt that this loss would be compensated for by what could be gained by engaging preservice teachers in sustained and focused reflection that was grounded in classroom-based evidence.

Adaptations for the Methods Course

During the methods course, annotated agendas, facilitator notes, PowerPoint® slides, and question prompts that are included with the LTLF facilitation resources provided the

core of the plan for instruction. A straightforward adaptation was reworking the agendas to accommodate the course schedule (two 110-minute class meetings per week). Two more substantial modifications are discussed below.

Connecting to Practice

The *Linking to Practice* pieces of the LTLF curriculum are designed for teachers to connect what they are learning in the sessions to their classroom practice. This is the one area of the curriculum that doesn't directly translate to working with preservice teachers, although their need to make connections between what they are learning and their own teaching remains critical. A long-standing component of the middle school methods course is a series of three group field experiences with an intensive reflection component; we modified this experience to capitalize on the preservice teachers' experiences in the LTLF curriculum (see Mumme & Seago, in development, and Van Zoest, 2004, for more details). The field experiences now take place later in the semester, after the preservice teachers have gained the experience of analyzing and discussing student thinking by completing the LTLF foundation module. During these experiences, each preservice teacher facilitates a small group of middle school students as they work to solve a mathematics problem selected directly from the LTLF curriculum. This ensures that the preservice teachers are very familiar with the mathematics involved in the problems and have studied different approaches students might take to solve them. These experiences are intended to help the preservice teachers further their understanding of how middle school students think mathematically and to provide a valuable opportunity for them to practice their questioning and listening skills in the context of a community that can both support and challenge them.

Required Readings

Although suggested readings are provided in the LTLF facilitation resources, they aren't an integral part of the curriculum. We incorporated readings from a range of mathematics education publications, including portions of the NCTM *Standards* (2000), throughout the LTLF foundation module (see Figure 1 for selected publications) and set the expectation that the preservice teachers would use the readings to inform our class discussions. Assigning these readings seems essential to the learning that took place in the course, as they gave the preservice teachers tools to begin to think about learning mathematics in ways other than how they had learned as students themselves, and language to discuss such learning.

Insert Figure 1 About Here

Preservice Teacher Learning

In the remainder of the chapter we turn our attention to what the preservice teachers learned as a result of the LTLF curriculum. We use classroom transcripts from the spring 2005 semester to focus on two key areas of learning that mirror the dual focus of the curriculum on mathematics and pedagogy.

Learning of Mathematics

We have found the LTLF curriculum to be particularly useful for increasing the preservice teachers' *mathematical knowledge for teaching*. This is what Ball and Bass (2000) have described as the mathematics that teachers do in the course of their work. We use their delineation of these tasks (Ball, Bass, & Hill, 2004) as a framework for the kinds of mathematics we want our preservice teachers to be learning. Figure 2 delineates some of the

ways in which we feel the LTLF curriculum provides opportunities for preservice teachers to develop mathematical knowledge for teaching. In the remainder of this section, we provide a specific illustration of the nature of this learning.

Insert Figure 2 About Here

Our example comes from LTLF Session 6: Regina's Logo. The following dialogue occurred when the preservice teachers were discussing their solution strategies after having already worked independently on the mathematical problem shown in Figure 3. The written work that was made public during these segments of discussion is provided in Figure 4.

Insert Figures 3 & 4 About Here

In response to a request for someone to share his or her thinking, Laurel¹ presented her approach:

First thing I did was I made a table to write down the size of each one, it goes to 4. Then, this was the simplest way, I wanted to break it down really simple. Then I just wrote the number of each block in each one. So I just counted it out, it was five, eight, eleven and fourteen. Then, I counted the middle blocks for how many were in the middle for each time. So in the first one there was one, the second one there was two, the third one there was three, the fourth one there was four and I noticed that this and this were the same [pointing to top and bottom rows of squares].

After some clarifications about what she meant in response to questions, she continued:

Okay. Then I counted the top and bottom, these sides. And in the first one there was two on the top then there was three on the top, then there was four, then there was five, so two, three, four, five. I noticed that this was one plus either one of these 'cause these are the same so I noticed that it was $n + 1$ to count for the top and the sides, the top and the bottom, but I know that there's not only a top but there's also a bottom so you want to multiply by 2 and then to account for the middle pieces you're always going to have to add that on as well and that's how I got it, in a nutshell.

At that point the instructor, asked, "Anyone have any questions for Laurel?" and Steven and Laurel had the following rapid interchange:

Steven: The only thing I see with that is like what if someone, another student, visualized the middle as like the entire thing [interrupted]

Laurel: Yeah, that's what I wrote in my paper, like that's another way to do it and you'd have to account for someone else [interrupted]

Steven: So basically that would switch the 2 to like $2n + 1 + n + 1$.

Laurel: Yeah, like you could explain it that way, but I see, like what you're saying is that there's more than one way to think of that problem.

At the instructor's request, Steven came to the board and elaborated on his variation of Laurel's solution strategy (see Figure 4). The class then discussed Laurel and Steven's methods and gave them suggestions for clarifying their methods that included this comment from Rick:

I think it would be better, instead of saying top and bottom for Steven's, wouldn't it be better to say left and right or something like that? Cause even if I didn't have the thinking of Laurel, I would be confused about top and bottom.

After some discussion among the class, the following ensued:

Nathan: [Draws the diagram in Figure 4 on the board] I called this $t + 1$ and this $t + 1$ so I didn't have to say, I didn't have these parts of the chart [pointing to columns], so I didn't have any confusion about the top and bottom. This is $t + 1$, this is $t + 1$, and then what's left is just t , and then I saw I had two of these, so I had $2(t + 1)$ and I had to add my extra t that was in there. And then looking at it the other way [referring to Steven's picture in Figure 4] I started with this and called this $t + 2$ and then each of these left over was t so then I had $2t + t + 2$. So that's here and that's that, it's just the same thing, but without the table like that, so I don't have to label my top and bottom or anything.

Instructor: So yours was more visual, just going right directly from the picture.

Nathan: Yeah, it just clears that up, there's no "what do we call the top, what do we call the bottom." Not as much confusion.

Instructor: Anyone have any questions for Nathan? Any comments? [pause] Does that help you think about it better? Is it the same as the table? What do the rest of you think about that?

Mike: I think it's a lot easier to see, personally, the diagram of it, it just seems to make a better explanation for, you know, the middle being just one, you know, the top and bottom being two, you see the information in the chart, but you don't

understand why it's one, two, that sort of thing. But with the drawing you can see like the top's $t + 1$, the bottom's $t + 1$, the middle's t , that sort of thing, so you're understanding all the reasoning, like all the numbers.

Although the above are only small segments of a rather lengthy discussion of the Regina's Logo task, it illustrates the way in which the preservice teachers were able to improve their mathematical explanations and make connections among different representations. Perhaps most importantly, we see a developing awareness of the value of a visual representation and the way in which it can be a tool to better understand the mathematics in the situation. This should not be underestimated, as a common misinterpretation of the call for multiple representations in the *Standards* is to have students rote generate tables, graphs, and symbolic representations with little or no discussion about the connections among them or sense of the benefits that the different representations offer.

As is typical, after the preservice teachers had discussed the mathematics in Regina's Logo, they watched the video clip of students discussing this task. While doing so, they made hypotheses about what they observed and then used the transcript to check the validity of their hypotheses and look for alternate interpretations. The instructor initiated a discussion about their reflections by asking about "mathematically interesting pieces of the video."

Thomas responded with:

I think the point where Alexis [a student in the video] talked about using the distributive property to make the equations equal, that one was just a simplified version of the other was important to show that she was recognizing that uh, each equation went at it a different way but it was really the same thing, that they were both getting to the same result and answer, even though they were looking at it a different way.

The discussion that followed included this interaction:

Instructor: Why was it important that she recognize that they were equal in this case?

Rick: Because there's two different methods of looking at the problem. You know, both of them can be represented with an equation, both equations can be manipulated to be visually equal, visually as well as algebraically equal.

Instructor: [after a pause] And we're supposed to be doing the building [referring to a classroom norm of building on each other's comments, as described in Sherin (2000)]

Reed: In taking that one step farther, she possibly recognizes that it's going to be easier to formulate an equation using the closed method instead of recursively, and then that will help with the next part of the lesson where it's, Schemel's Logo [the next problem in the LTLF curriculum, which the preservice teachers had completed as homework], which it looks like it would be pretty hard to figure out recursively.

Here we see that, with prompting, the preservice teachers are developing an understanding of what it means to acknowledge students' mathematical thinking, make sense of it, and determine whether it is likely to be productive. Furthermore, they are engaging with mathematical ideas (e.g. distributive property, recursion, equality) that are fundamental to their future students' learning.

One thing that has become clear to us through watching our students engage in these opportunities to develop mathematical knowledge for teaching is the difficulty of both developing and implementing such knowledge. One of the advantages of the LTLF curriculum is that it is a coherent curriculum and is designed to develop understanding over time. Even with this, we have seen through the field experiences that implementing this knowledge while teaching requires another level of skill and expertise—one that is best developed through continued interaction with students. It is because of this ongoing nature of the development of mathematical knowledge for teaching that it is critical that preservice teachers develop a reflective stance for teaching—one that will enable them to learn from their ongoing teaching experiences.

Development of a Reflective Stance

We have documented three important shifts in how preservice teachers who engaged with the LTLF curriculum analyzed instances of practice: (1) a shift from evaluating

instructional decisions based on affective measures to analyzing instruction based on pedagogical considerations; (2) a shift from knowing about student thinking to conjecturing about student thinking; and (3) a shift towards considering how instructional decisions might affect student thinking (Stocker, 2006). Each of these shifts is illustrated and briefly discussed below.

Shift from affective to pedagogical considerations. Early in the semester, the preservice teachers tended to consider instructional decisions based on affective measures, as is seen in the following excerpt from LTLF Session 1: Growing Dots, in which Mandi discusses Kirk's (a teacher in the LTLF video) decision to push a student (James) to go to the board to explain his solution:

I don't think that the teacher should have like made him go up there. Especially at that time, like kids are just in adolescence and they're like, it's a rough time and they don't want to be put on the spot in front of the whole class. So like, he didn't want to go up there and the teacher was like, big deal, come up here anyway. Like, I don't think he should have made him come up if he didn't want to.

In this passage, Mandi is disagreeing with Kirk's decision based on how it might have made James feel, rather than considering pedagogical reasons that Kirk may have had for encouraging James to share his thinking. For example, James had solved the problem recursively and had arrived at a different solution than a student who had shared her explicit expression for solving the same problem—raising important mathematical issues that Kirk may have wanted other students to think about.

Throughout the semester, the preservice teachers increasingly began to consider pedagogical and mathematical reasons for instructional decisions. For example, in a discussion of LTLF Session 6: Regina's Logo, Cathy comments on a teacher's (Gisele) less central role in her classroom. In the video clip, Gisele is standing off to the side of the

classroom, while two students are at the board recording other students' solutions and asking questions of their classmates.

I just think she takes it to, like, too much of an extreme. If I was a kid who didn't understand, I would tune out...I don't know which one knows what they're talking about, so I'll just tune out until we get to the end and maybe somebody will explain what's going on.

Although Cathy's response is a reaction based on how she might feel as a student, it also shows some consideration for student learning. More significantly, other preservice teachers responded to Cathy's comment by offering opposing opinions that were based on pedagogical considerations, such as the comment by Thomas below.

If you need your teacher to constantly tell you at what point the mathematical idea is important, at what time do you move from the teacher being the one of authority to maybe one of your peers, one of the students coming up with an idea that's important and recognizing, on your own, that that's a good idea, an important mathematical concept? I don't know if the teacher is always saying "well this is important, this is important," that you would begin to make your own growth and be able to identify, by yourself, the important mathematical concepts, which I think is important as you get later in education and more into the real world.

Counter-responses like that made by Thomas did not occur early in the semester, and thus, this shift from interpretations of teacher moves based on affective measures to those based on pedagogical considerations represents an important change that took place during the class discussions.

Shift in interpreting student thinking. Early in the semester, the analyses of student thinking tended to be absolute in nature—not fully considering the reasoning behind a students' inaccurate or incomplete response, yet using definitive language. Later in the semester, however, the preservice teachers became more tentative in their analyses of student thinking.

Consider, for example, the following dialogue from LTLF Session 1: Growing Dots focused on James' thinking. James was considering the number of dots in a pattern that began with a single dot in the center and added four dots around it each second.

Rick: Didn't James get 400 instead of 401. So he just added wrong then, basically?

Mike: Yeah.

...

Rick: So, that's what he did though, right? He just simply added wrong? He just looked for, I mean, he probably didn't have enough time to add those...

Theresa: to 400, he probably just saw that he didn't have the one like she did and said like, "oh, well I got 400". I mean I think it would have taken him a long time to add to...

Rick: Right, exactly, that's the point.

Theresa: I mean he didn't do it all the way out, he just said, "Oh, well I'm wrong because I got 400."

In this segment, Rick assumes that James came up with an incorrect answer of 401 because of an adding error and both Mike and Theresa support his analysis. In the video, however, James clearly explained why he got 400 dots instead of 401 like his classmate: "'Cause she counted the center, we didn't count the center like she did." This type of analysis is characteristic of early in the semester—instead of turning to evidence from the transcript, these preservice teachers made ungrounded assumptions about James' thinking that were not countered by others in the class.

Discussions of student thinking in the LTLF videos became much richer and more grounded later in the semester, particularly after class discussions centered on a portion of the *Standards*, as well a number of articles that discussed classroom discourse. The excerpt

below from LTLF Session 6: Regina’s Logo illustrates this change. This dialogue builds on conjectures that had been shared about a student’s (Jordan) understanding:

Laurel: Ok, so like maybe he just got confused, what, like, his wording, his vocabulary at times meant...[the teacher] should have just like kind of slowed him down and then like, “Think about your vocabulary. You know, are you using the right words to describe what you’re trying to get across?”...

Mandi: That relates [to] the discourse articles that we read, like how the whole mathematical language and, like there’s the one example that was $2x$ and she kept saying x squared; it was really two x and the teacher was saying to them you have to say $2x$, not x squared....

Mike: I don’t think it has to do with vocabulary, I think it has to do with him just misunderstanding how to do it because like he talks about, you know, at first he says three x is x times x times x and then later on he also says that x plus two is equal to two x .

An important difference between this discussion and the discussion of James’ thinking is that instead of immediately concluding that Jordan didn’t understand the mathematics in the problem, the preservice teachers considered an alternative explanation—language—for the errors in Jordan’s thinking. In addition, they challenged each other’s ideas, rather than simply accepting the first explanation offered. This suggests that the preservice teachers were moving away from a definitive stance of *knowing* about student thinking to a more tentative stance in which they *made conjectures* based on the evidence available. As Ball and Cohen (1999) have argued, this tentative and inquisitive stance is a central quality of teachers who continually learn as they teach, and thus are able to adapt their teaching in response to students’ understandings.

Considering how instructional decisions affect student thinking. In early class discussions, the preservice teachers tended to talk about what they “liked” or “didn’t like” about the instructional decisions that a teacher made, as opposed to discussing how those decisions affected student learning. This changed, however, later in the semester. Consider,

for example, an excerpt from a class discussion following a field experience in which Hillary considers how her classmate's instructional decision might have hindered student thinking. In this case, the teacher, Vince, had introduced the term *variable* early in his session with the students.

I think also on doing it that way, Evan might have been a little bit thrown off by the teacher saying “let’s put a variable in there” if he wasn’t ready for that yet. Cause on line thirteen, Vince says, “Let’s just throw in the variable t. What’s t?” You know and Evan is still trying to explain, but he wasn’t thinking variables, he was thinking well I’m just doing top, bottom, and sides. I don’t know what you mean, you know... So that could have been maybe a little bit of Evan’s frustration at the beginning cause he wasn’t sure how to express it in variables. He just knew how, what he was seeing.

Rather than simply stating that she didn’t like the teacher’s actions, Hillary is reflecting on how an instructional decision affected student thinking, providing an example of the increased number of preservice teachers’ reflections that were focused on pedagogy and student thinking. This, along with the rest of the dialogue in the preceding sections provides evidence of important shifts toward an increased focus on student thinking that took place using the LTLF curriculum (see Stockero, 2006, for more detailed analyses). These shifts are significant, as it has been found that teachers who become more aware of student ideas through the use of videos subsequently pay more attention to such ideas in their own teaching practice (Borko & Putnam, 1996; Sherin, 2000, 2004).

Conclusions

The preservice teacher learning reported here suggests that curriculum materials that have been developed for use with practicing teachers can be effectively used at the preservice level. In particular, the LTLF professional development materials provided a coherent curriculum that allowed us to strengthen the course's main focus—teaching for student understanding by accessing and building on student thinking. Prior to using the LTLF video

curriculum, the middle school methods course had used a collection of cases drawn from a variety of sources. Although we focused class discussions on the student thinking in each case, it was difficult to connect the cases together so that they built on each other to form a coherent curriculum. The LTLF cases solved this dilemma because they were intentionally designed to connect and build on each other both mathematically and pedagogically.

It is important to remember, however, that in this situation the learning goals for the course and the curriculum materials were compatible, and the curriculum was used intact. Making substantial adaptations to the curriculum, such as using only some of the video segments, would require careful planning and consideration of the consequences of such adaptations. The preservice teachers we worked with did not begin to show significant changes in their reflection until a month into the LTLF curriculum (Stockero, 2006), suggesting that the sustained nature of the engagement was critical.

We conjecture that the adaptations we did make—incorporating field experiences and requiring readings—were essential in supporting the preservice teacher learning we have seen. With limited experiences to draw on, providing alternative perspectives via course readings was an important scaffold in the preservice teachers' learning and needs to be considered when adapting materials designed for inservice teachers for use at the preservice level. Furthermore, structuring the field experiences so that they built on the video cases—mathematically and in terms of the reflection that was required—also seemed to support the preservice teachers' first attempts at reflecting on their own practice in a way that allowed them to recognize both their strengths and their areas for continued work.

The challenging nature of teaching makes it essential that teachers enter the field with a foundation of mathematical knowledge for teaching and a reflective stance that allows them

to learn from their ongoing teaching experience. Our work provides support for the use of a video-case curriculum as a means of both developing a reflective stance in preservice teachers and furthering their understanding of the mathematics needed for teaching.

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Figure 1: Selected Readings Assigned During LTLF Component of Methods Course

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- Pirie, S.E.B. (1998). Crossing the gulf between thought and symbol: Language as (slippery) stepping-stones. In Steinbring, Bussi, and Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 7-29). Reston, VA: National Council of Teachers of Mathematics.
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Figure 2: The Mathematical Work of Teaching Engaged in through the LTLF Curriculum

| | |
|---|--|
| <p>Mathematical Work of Teaching (Ball, Bass, & Hill, 2004, p. 59)</p> | <p>Opportunities to Engage in this Work Offered by the LTLF Curriculum (as Adapted for Preservice Teachers)</p> |
| <ul style="list-style-type: none"> • Design mathematically accurate explanations that are comprehensible and useful for students | <p>These are addressed through the mathematics problems in the LTLF curriculum:</p> |
| <ul style="list-style-type: none"> • Use mathematically appropriate and comprehensible definitions | <ul style="list-style-type: none"> • Doing mathematics from the videocases as learners • Discussing their work in the university classroom with high expectations for quality and thoroughness • Focusing on multiple representations and the connections between them |
| <ul style="list-style-type: none"> • Represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process | |
| <ul style="list-style-type: none"> • Interpret and make mathematical and pedagogical judgments about students' questions, solutions, problems, and insights (both predictable and unusual) | |
| <ul style="list-style-type: none"> • Be able to respond productively to students' mathematical questions and curiosities | <p>These are practiced through engaging with the videocases in the LTLF curriculum and applied in the field experience component:</p> |
| <ul style="list-style-type: none"> • Make judgments about the mathematical quality of instructional materials and modify as necessary | <ul style="list-style-type: none"> • Watching the videotapes of classroom teaching, using the transcripts to follow-up on hypotheses, and generating hypothetical next moves • Planning a lesson, teaching the lesson, reflecting on what they have learned, and modifying the lesson prior to teaching it to another group of students • Teaching a second lesson to the same group of students • Maintaining high standards of explanation and evidence during class discussions of both the videocases and their teaching experiences |
| <ul style="list-style-type: none"> • Be able to pose good mathematical questions and problems that are productive for students' learning | |
| <ul style="list-style-type: none"> • Assess students' mathematics learning and take next steps | |

Figure 3: Regina's Logo Problem

Assume the pattern continues to grow in the same manner. Find a rule or formula to determine the number of tiles in a figure of any size.

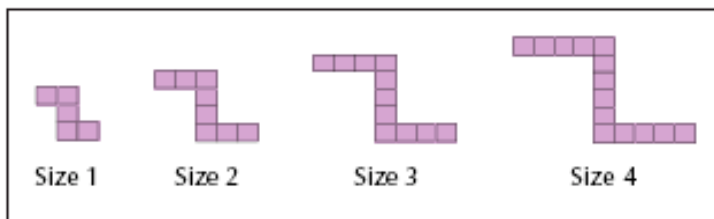
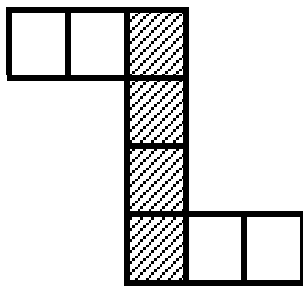


Figure 4: Preservice Teacher Work from Regina's Logo

Laurel

| size | # blocks | middle | top (or bottom) | top & bottom |
|------|----------|----------------|-----------------|--------------|
| 1 | 5 | 1 | 2 | 4 |
| 2 | 8 | 2 | 3 | 6 |
| 3 | 11 | 3 | 4 | 8 |
| 4 | 14 | 4 | 5 | 10 |
| n | | n | n + 1 | 2(n + 1) |
| | | $2(n + 1) + n$ | | |

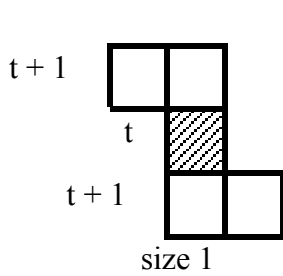
Steven



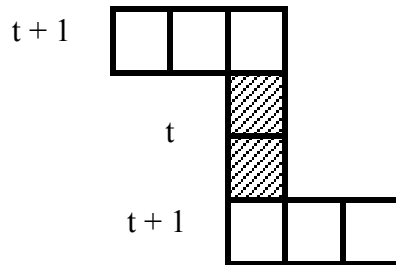
size 2

| size | middle | top & bottom |
|------|--------|--------------|
| 1 | 2 3 | 2 |
| 2 | 3 4 | 4 |
| 3 | 4 5 | 6 |
| 4 | 5 6 | 8 |
| | n + 2 | 2n |

Nathan



size 1



size 2

¹All names are pseudonyms except for those in the LTLF videos.