Presentation at the *Second International Mathematics Curriculum Conference*

Chicago, 2008

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[...] Consider for example justification and argumentation, these are disciplinary practices in mathematics, but in school mathematics these are learning practices. In mathematics justification and argumentation are disciplinary practices because they are the means by which mathematicians validate new mathematics. In school mathematics argumentation and justification are learning practices because they are the means by which students enhance their understanding of mathematics and their proficiency at doing mathematics.

ICMI Symposium on the Occasion of the 100th Anniversary of ICMI
Working Group 1: Disciplinary Mathematics and School Mathematics

Why are Disciplinary Practices in Mathematics Important as Learning Practices in School Mathematics?

Terry Wood, Purdue University
Megan Staples, University of Connecticut
Sean Larsen and Karen Marrongelle, Portland State University

I agree with Wood, Staples, Larsen and Marrogelle:
disciplinary practices in mathematics, are not the same as learning practices in mathematics education.
Excellent point.

But I wonder whether normal people would make this kind of consideration.

In this paper I argue that school mathematics is not, and perhaps never can be, a subset of the recognised discipline of mathematics, because it has different warrants for truth, different forms of reasoning, different core activities, different purposes, and necessarily truncates mathematical activity. [...] The relationship of school mathematics to adult competence is similar to the relationship [...] between being made to eat all your spinach and becoming a chef; between being forced to practise scales and becoming a pianist. [...] That some people become [...] beautiful pianists or inspiring cooks is interesting, but what is more interesting is the fact that most people who go through these early experiences do not: instead they merely follow orders, or hate green vegetables, or give up practising their instruments.

ICMI Symposium on the Occasion of the 100th Anniversary of ICMI
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School mathematics as a special kind of mathematics
Anne Watson
University of Oxford

I quite agree with Anne Watson: *school mathematics is not, and perhaps never can be, a subset of the recognised discipline of mathematics*.

But I wish to add a few other examples:

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What about learning to wear clothes?

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What about learning to eat using cutlery?

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What about learning to wear clothes?

What about learning to eat using cutlery?

What about learning not to make noise in some places (eg, churches) and learning to ***do*** *make noise in others (eg, (real) football matches)?*

When that happens,

When that happens,

Is anyone concerned with everybody becoming a clothes designer?

When that happens,

Is anyone concerned with everybody becoming a clothes designer?

Is anyone concerned with everybody becoming a gourmet?

When that happens,

Is anyone concerned with everybody becoming a clothes designer?

Is anyone concerned with everybody becoming a gourmet?

Everybody is *concerned with people being respectful of where they are as socially agreed...*

Teacher training in mathematics involves much more than just learning how to manage a classroom effectively. […]. Mathematics teachers are passing on values, habits and customs as well as knowledge and skills. They are inducting their students into the culture of mathematics.

Paper presented at ICMI Regional Conference, Shanghai, China, July 1994.
Educating the mathematical enculturators
Alan J. Bishop

I *completely* agree with Alan Bishop.

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And I have once even considered
the role of a teacher as, in a sense,
similar to that of a tourist guide.

(Monster Theory; the mathematician’s garden)

But there is a problem:
What is indeed *the culture of mathemathics*?

How is it to be characterised:
through its deep structures
or through its superficial, *visible*, features,
or both?

I would like to make clear
that I highly respect the work
on early algebra
done by the people I mentioned and many others, many of whom are here today.

But I want to consider *something else*
that is, in my view, still missing:

What is it that normal people *see* when they see us ‘doing algebra’? What does it look like ‘doing algebra’ from the point of view of normal people?

That will give us one *missing* side of the story.

What do we *wish* people to accept with respect to the *looks* of 'doing algebra'?

That will give us the other side of the story.

And unless we have both sides,
quite likely we will — again —
be answering question they (normal people)
did not ask us and would probably
never ask us...

As to the first side, a longish but worthy quote (my translation):

I have always resisted what can be called 'social tourism', that kind of tourism that encompasses visiting shanty towns, indigenous communities or 'popular' neighbourhoods. It disgusts me the way in which we have converted poverty into something photogenic, something that goes well into our photo albums that we happen to show to others at the end of our trips. I am disgusted by the kind of reports that this kind of tourism usually produces, those reports full of surprise, arrogance, good consciousness and good will.

I can't avoid the sensation that in our trips to misery we look for pictures to show, stories to tell, hunting trophies to show off. But, despite all my caution, there I was, under those black plastic sheets made into homes, accepting the drink offered by a woman whose name I can't remember, asking her question after question about her way of living, her family,

how she got there, her difficulties, her projects, her hopes. As I was a little ashamed by the impunity for asking her any question, including the most personal ones, the most indiscrete, I went into imagininig the impossibility of a reverse situation: someone from the LWM [Landless Workers Movement] that knocks on my door, declaring an interest on the life stories of male heterossexual university professors of my generation, that, stunned, browses through my house, that helps her/himself to a glass of wine and that feels totally at ease to ask me whatever about my way of life, my personal trajectory, my expectations, my ideas, my successes, my loves, my failures, my happiness, my sadness, telling me that s/he will publish something about my personal experiences on a collectively published book by poor and illiterate researchers interested on the ways of life of subjects who are rich and have reached university level schooling and profession.

By Jorge Larossa (Spain):

I thank him for having said that.

The key point here seems to be:

what we do, the way we think about
our children's thinking and our views
on what we want them to learn

are not quite what they do,
not quite the way they think about their thinking,
not quite what they believe they have to learn.

Much as

school mathematics is not, and perhaps never can be, a subset of the recognised discipline of mathematics

disciplinary practices in mathematics, are not the same as learning practices in mathematics education

Normal people do not distinguish *deep* from *superficial structure*.

Normal people do not distinguish *deep* from *superficial structure*.

Normal people do not distinguish *procedural* from *relational thinking*

Normal people do not distinguish
deep from *superficial structure*.

Normal people do not distinguish
procedural from *relational thinking*

It’s not that they are unable to do it:
this is simply not they way normal people
organise their world.
(categories)

But normal people

do organise their experience by *describing* what we choose to call *the 'superficial' features of it*

and

do adapt to what *seem* to be the rules governing life in given contexts.

Is it so surprising
that they come to believe,
after *years and years*
using numbers to calculate
and letters to write,
that algebraic manipulation
is akin to
things crazy people do?

(as said by my dearest friend Zezé,
who is in charge of general services where I work,
and who regularly *sees*,
in the coffee room's blackboard,
what my son once called, when he was 9, *alien language*):

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$

At this point, finally,

the first slide:

The unbearable lightness of being (superficial)

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The starting point is:

As soon as *they* say:

“why do i have to eat spinach?”
“why do i have to wear clothes?”
“why minus times minus is plus...”

Our job is beginning to become harder...

As much as we can induce them (culturally)
into wearing clothes (superficial?),
we can induce them (culturally)
into *accepting* that mixing letters and numbers
is *legitimate* (superficial?).

Nothing beyond that.

No deep structure immediately intended (not that we should not work on that!).

Perhaps nothing we would be proud of
in our conference paper presentations...

My central point is:

The failure of our students in algebra
is not the failure
*of those who **tried** and failed.*

It is rather, I argue,
the failure
of those who never tried to succeed in it.

(because what they see *is not* legitimate;
that does not *make sense*)

(MONSTER THEORY according to Lins)

So,

let’s identify what ***normal*** *(*big and small) people see in *doing algebra;*

let’s identify what ***we*** see in doing algebra

and

let’s create opportunities for our *young* children to become *used to certain* ***values of doing algebra as we see it****.*

And this has to do with them *accepting* at least taking a look at what we are saying.

Hands-on!!

A shop sells packages of snacks and soda cans.
You cannot buy separate snacks or cans in this weird shop...

They sell the following packages:

Package A: 1 snack and 3 cans
Package B: 2 snacks and 3 cans
Package C: 2 snacks and 4 cans
Package D: 2 snacks and 6 cans
Package E: 3 snacks and 5 cans
Package F: 4 snacks and 3 cans

Package A: 1 snack and 3 cans
Package B: 2 snacks and 3 cans
Package C: 2 snacks and 4 cans
Package D: 2 snacks and 6 cans
Package E: 3 snacks and 5 cans
Package F: 4 snacks and 3 cans

How can one buy:

3 snacks and 6 cans?
5 snacks and 8 cans?
1 snack and 1 can?
6 snacks and 9 cans? (two different ways)

And so on.

Package A: 1 snack and 3 cans
Package B: 2 snacks and 3 cans
Package C: 2 snacks and 4 cans
Package D: 2 snacks and 6 cans
Package E: 3 snacks and 5 cans
Package F: 4 snacks and 3 cans

How can one buy:

3 snacks and 6 cans?
5 snacks and 8 cans?
1 snack and 1 can?
6 snacks and 9 cans?

And so on.

What is ‘algebraic’ about this?
Not much, depending on the eyes of the beholder.

Much, in my view.

Operating on expressions *as whole objects:*

3 snacks and 6 cans = (1 snack and 3 cans) + (2 snacks and 3 cans)

which easily turns into

3 S and 6 C = (1 S and 3 C) + (2 S and 3 C)

and into

$$3\ S + 6\ C = (1\ S + 3\ C) + (2\ S + 3\ C)$$

perhaps with a little help from the teacher
(but not to Vigotsky’s opposition!)

And there we are:

second to fourth graders
mixing letters
with numbers
with other mathematical signs.

Working with polynomials!

And they only had to do it once
to ***learn*** that this is *legitimate*.

Around 300 children in Brazil,grades 1 to 4,
‘post-tested’ a few months later
through the activity that follows
(the same but not quite):

Daniel is helping his aunt, who owns a records shop (she went on a boat trip). When he gets to the shop, Monday morning, he realises that he knew his aunt sold the records for a single price, and the tapes for a single price, too. But he forgot to ask what the prices were!

Looking around he found a piece of paper with some of Friday's ***sales***: (notice my didactical emphasis...)

1 record and 5 tapes – R$ 65
3 records and 4 tapes – R$ 85
2 records and 1 tape – R$ 40
4 records and 3 tapes – R$ 90
5 records and 2 tapes – R$ 95

Customers are arriving!!

Let’s help Daniel to calculate the cost of some new sales!

1 record and 5 tapes – R$ 65
3 records and 4 tapes – R$ 85
2 records and 1 tape – R$ 40
4 records and 3 tapes – R$ 90
5 records and 2 tapes – R$ 95

4 records and 9 tapes
4 records and 2 tapes
3 records and 1 tape
1 record and 1 tape

What is more expensive: a record or a tape?

Teachers suggested (and pupils promptly accepted and used) the following notation:

1 record and 5 tapes = R$ 65
\+ 3 records and 4 tapes = R$ 85
4 records and 9 tapes = R$ 150

What's in it:

Operating *with* and *on*
expressions as whole objects
(Adil Poloni, master's thesis)

Same 'experience' with notation as before
(Students 'imported' the notations themselves)

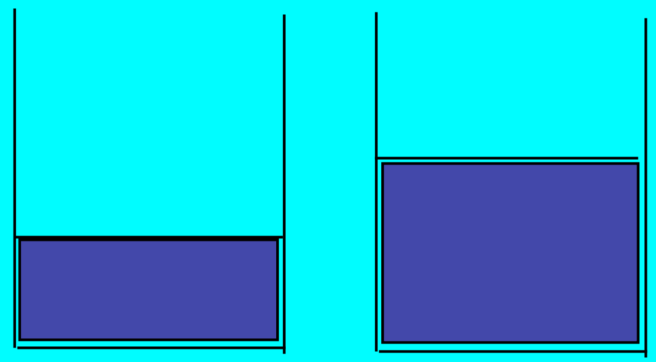
More operations: +, -, x, ÷
(logic of the operations: Model of Semantic Fields)

The importance of looking at the surface:

literally *mixing* numbers, letters, +, -, x, ÷, =, ()

packages holding *expressions* together
sales holding *expressions* together

The tanks



These two tanks are identical.

The tank on the left needs another 9 buckets full of water to fill it up.
The tank on the right needs another 5 buckets full of water to fill it up.

What can we say about this situation?

Let’s agree on:

calling the amount of water on the left tank X
calling the amount of water on the right tank Y
(6th grade students’ suggestion)

calling the bucket b
(teacher’s suggestion)

X + 4b = Y
"because if you *add* 4 buckets on the left there will be only 5 buckets missing"

Y – 4b = X
"because if you *remove* 4 buckets from the right there will also be 9 buckets missing"

Y is more than X
"because you can see from the drawing"
"because less is missing on the right"

X + 2b = Y – 2b
"because 7 buckets will be missing on both sides"

X + 5b = Y + 1b (notice: 1b, not b!)
"4 buckets missing on each side"

X – 2b = Y – 6b
"11 buckets missing on each side"

And then things start to look interesting:

$X + 6b = X + 2b$
“because, because... Oh, no, it’s not X, it’s Y...!”

Teacher: what could make this statement true:
Student: “If the bucket didn’t have a bottom!!”

$X + 20b = Y + 20b$
“anything over 5 and 9 buckets will overflow the tanks!!”

Teacher: “$X – 50b = Y – 54b$”
Students: “it’s impossible to do that!! You can see from the drawing...”

$Y – X = 4b$
“because if you *remove* from Y the same amount that’s in X...”
But how can one *do* that?

But the aim was not on mixing letters and numbers (useful),
nor was it on making relations (very useful):

the aim was to produce a set of statements
that are *something* ***for them***,
that are *legitimate* ***for them,***

so *they* can operate on them *as objects*:

Teacher:

“How would you *DESCRIBE*
the direct transformation ((naming of action)) of

$X + 4b = Y$ (legitimate statement)

into

$X + 5b = Y + 1b$ (legitimate statement)?”

Students:

“Add 1 bucket to each tank...”

And from there, with a new *legitimate* action, we could move to
generating new legitimate statements
offering both “tanks-based” and “direct transformations” justifications.

Y – 5b = X – 1b

either

“6 buckets missing on each side”

or

“remove 1 bucket from each side of Y – 4b = X”

To

source, target, transformation:

"Transform X + 4b = Y to make it look like b = ..."

To

source, target, transformation:

“Transform X + 4b = Y to make it look like b = ...”

That was homework. The next day, two answers:

$$b = (Y - X)/4$$

and...

To

source, target, transformation:

"Transform $X + 4b = Y$ to make it look like $b = ...$"

That was homework. The next day, two answers:

$$b = (Y - X)/4$$

and...

$$b = Y - X - 3b$$

The generality of the idea is:

here are we in front of a *situation: what can we do say about it*?
(whatever the situation is)

The idea serves biology as well as maths and language and story telling: *what you say about it*?

That is why it is not *disciplinary.*
That is why it fits so nicely into
K-6 classroom work:
interdisciplinary, cross-disciplinary, trans-diciplinary,
a-disciplinary work.

Who knows whether a student will say
"people who owns the left tank will have to be
more careful with their water use...!"

But there are specific aspects that interest us, for instance:

there was nothing in the whole process
that the students were not able to do before.
NOTHING.
What have they learned, then?

They have learned that they can do all that
in mathematics: legitimacy.

They have learned that expressions can be directly transformed
following rules that *legitimately* (for them)
apply to those (*legitimate)* expressions.

They have learned that transforming an expression
to put it into another, given *form*,
is something people do in some situations.
(*legitimacy*).

And so on.

Just to summarise before I finish:

We, in our research group, are interested in looking at the surface
from K to university
(for instance, what is there of ‘dimension’
in a real vector space whose vectors are R^2 and has dimension 3:
work of Amarildo da Silva and Rejane Julio)

and

we are interested in *dealing with superficiality*
in an *explicit and intentional way*
as a strategy to promote value internalisation
(students being internalised by values),
as early as possible.

That, we argue,
will give them a chance to at least *try*
the deep structure thinking we are willing to introduce them to.

And we are also interested
in developing an approach to teacher education and development
that is based on
categories that organise everyday life
(for instance, natural space, decision making)
categories that are bound to be present
in school classrooms.

And starting with them, ask questions like:
“what changes if we introduce this or that
‘mathematical’ idea or tool?
What can we say now that we could not before?”

Let’s take culture and cultural values *seriously*:

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why is it that forcing women to use burkhas,
is unacceptable,
under the risk of being punished if they don’t

Let’s take culture and cultural values *seriously*:

why is it that forcing women to use burkhas,
is unacceptable,
under the risk of being punished if they don’t

but forcing women to cover the upper body
or most of it, breasts mandatory in most cases,
is not?

Let’s take culture and cultural values *seriously*:

why is it that forcing women to use burkhas,
is unacceptable,
under the risk of being punished if they don’t

but forcing women to cover the upper body
or most of it, breasts obligatory,
is not?

Is this a superficial matter that we should not worry about?

Thank you.